

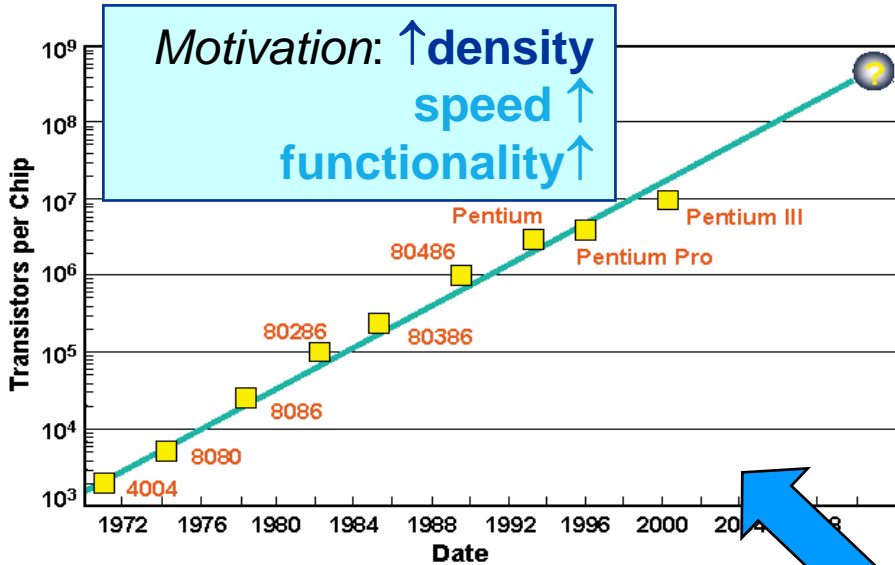
Computation vis-à-vis Physics: A Framework

VIA 2020 Forum, July 10 & 11, 2008

Ralph K. Cavin, III, Victor V. Zhirnov & Sadasivan Shankar (Intel)

- ◆ Relations between maximum computational performance and device physics
 - ❖ Correlations between computer performance and technology capability
 - ❖ Von Neumann threshold
- ◆ Binary switch abstraction
 - ❖ Generic floorplan and energetics
 - ❖ Connected Binary Switches
 - ❖ Floorspace, Timing and Energy for Communication between Binary Switches
- ◆ ‘Minimal’ Turing Machine
 - ❖ System scaling limits
 - ❖ Energetics and efficiency

Moore's Law: Binary Information Throughput



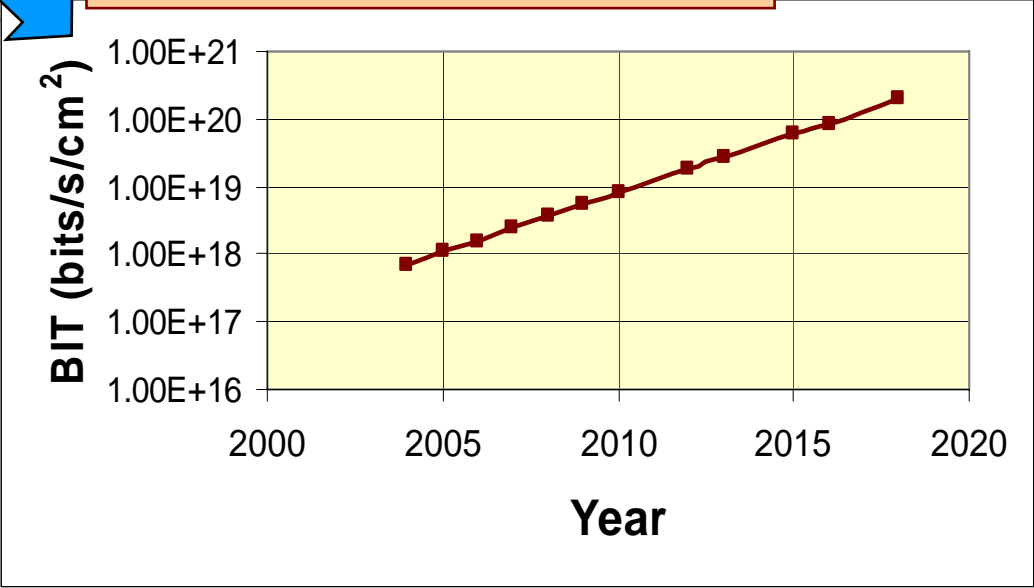
What is the ultimate number of binary transitions per second in a 1cm² chip area?

$$\beta = n_{bit} f$$

BIT

- a measure of computational capability on device level

n_{bit} – the number of binary states
 f – switching frequency



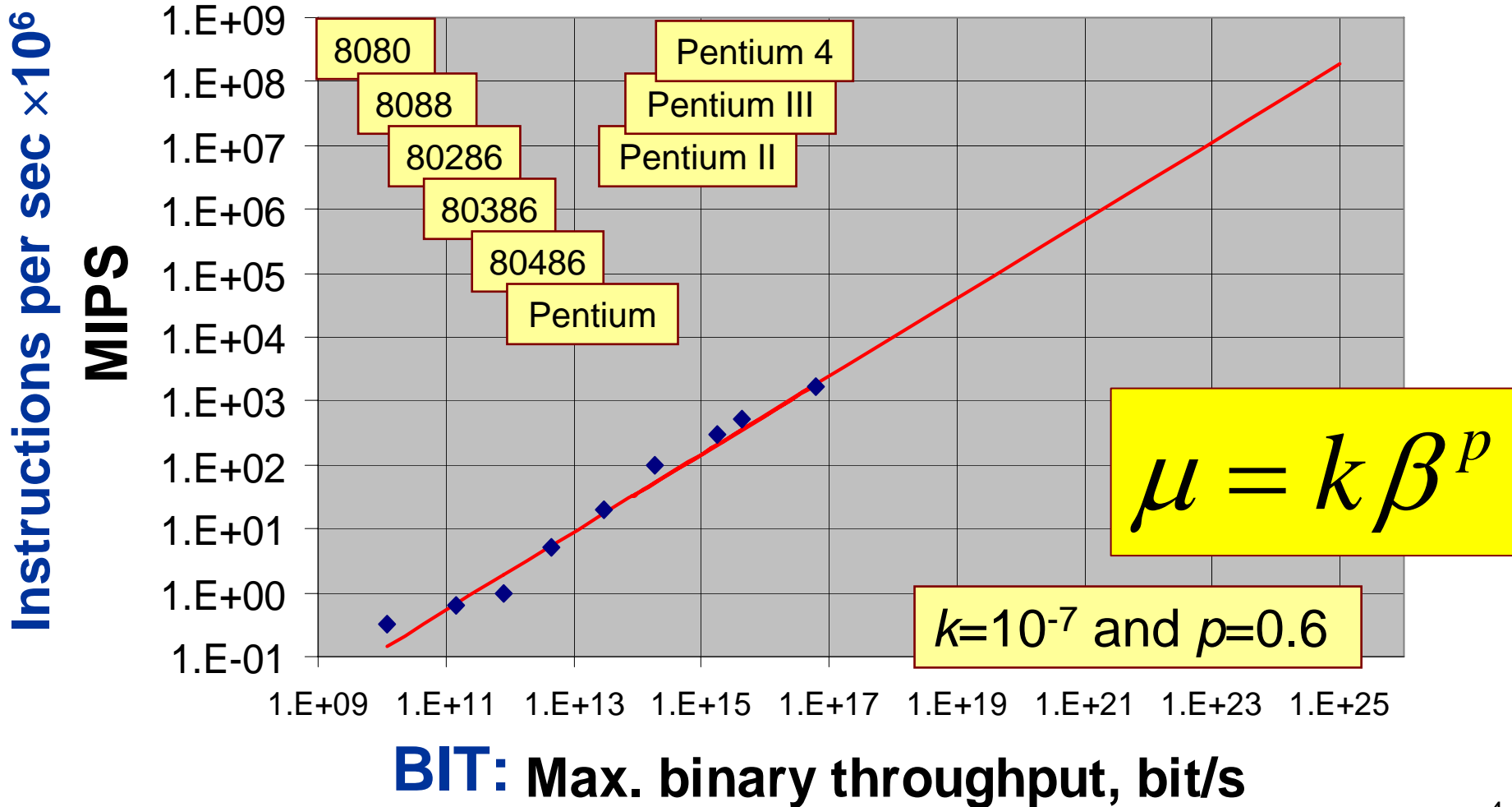
Why scaling? – To increase the *Binary Information Throughput* (BIT)

Source: Stan Williams, Hewlett Packard

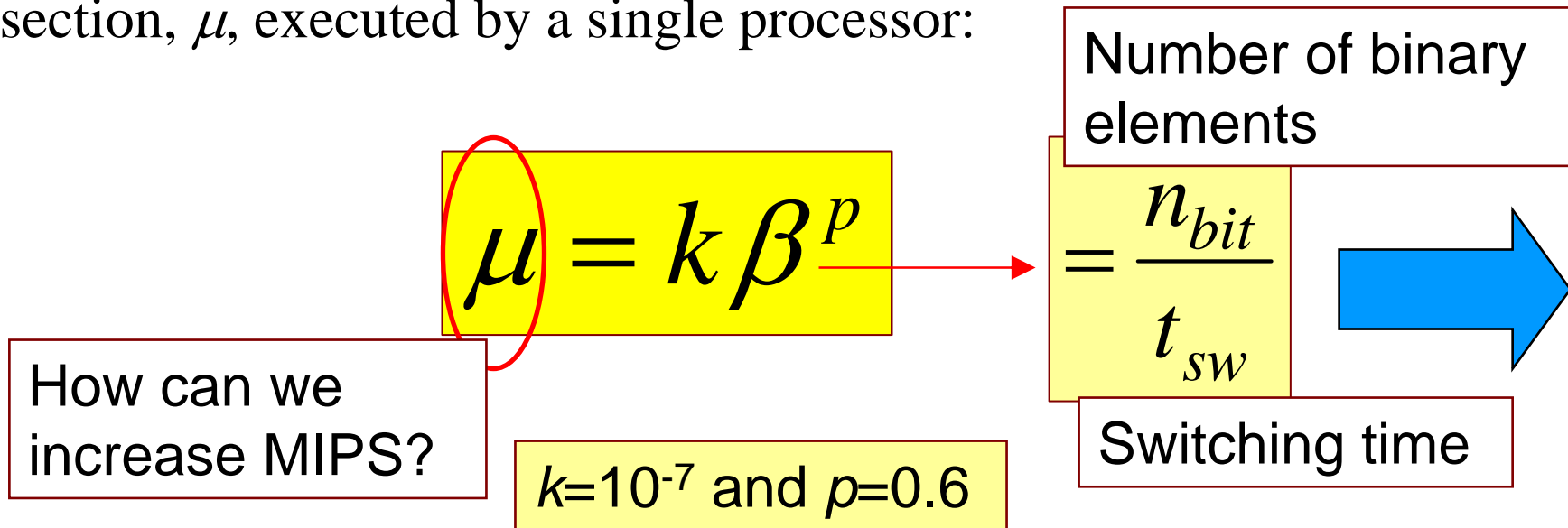
Computing Power: MIPS (μ) vs. BIT (β)



Sources: *The Intel Microprocessor Quick Reference Guide* and *TSCP Benchmark Scores*

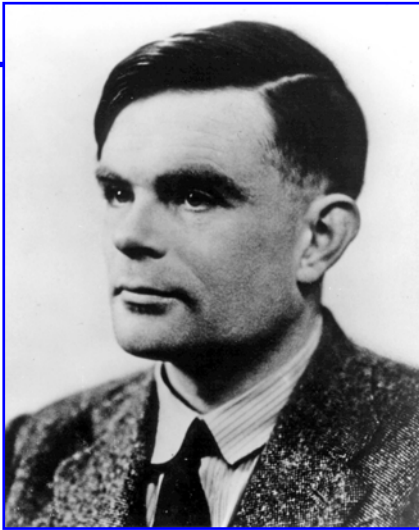


There appears to be a functional relationship between ultimate technology capability defined as the maximum number of binary transitions per unit time, β , and the millions of instructions executed per section, μ , executed by a single processor:



Turing-Heisenberg Rapprochement?

Instructions per second
a measure of computational
capability on the processor
level



Alan Turing

$$\mu = k \beta^p$$

**Binary Information
Throughput**

a measure of
computational capability
on device level



Werner Heisenberg



Ludwig Boltzmann

Can computational theory suggest new devices?
Stan Williams @ Nanomorphic Forum



We think that all devices operating in an equilibrium with thermal environment are governed by these relations, no matter what state variables are chosen!



$$\Pi_{error} = \exp\left(-\frac{E_b}{k_B T}\right)$$

$$\Delta x \Delta p \geq \hbar$$

$$\Delta E \Delta t \geq \hbar$$

“Boltzman constraint” on minimum switching energy

“Heisenberg constraints” on device size and speed

Nanoscale Devices

$$E_b^{\min} = k_B T \ln 2$$

$$x_{\min} = \frac{\hbar}{\sqrt{2mkT \ln 2}}$$

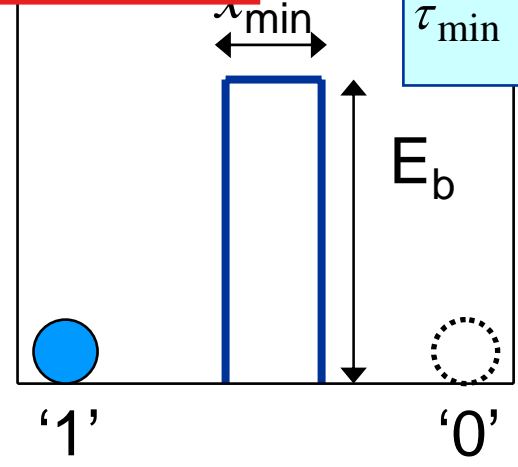
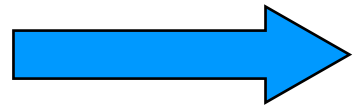
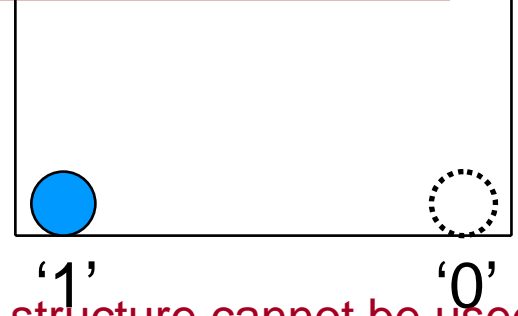
~ 10⁻²¹ J

~ 1.5 nm

$$E_{sw}^{\min} = 3k_B T \ln 2$$

$$\tau_{\min} = \frac{\hbar}{kT \ln 2}$$

~ 40 fs

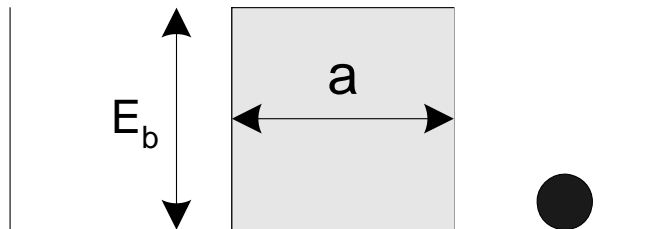
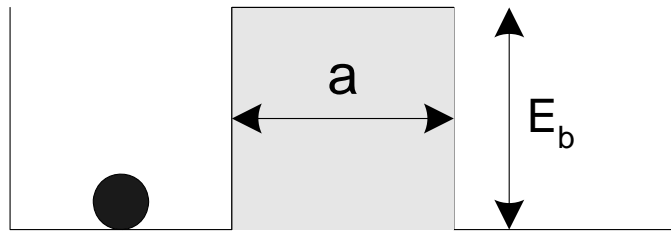
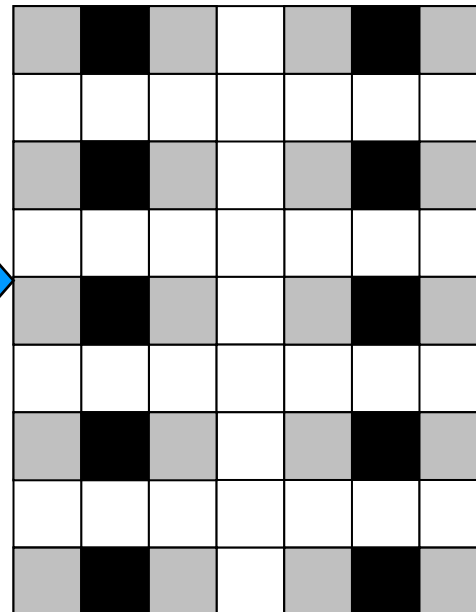


This structure cannot be used for representation/processing information

An energy barrier is needed to preserve a binary state

Two-well bit – Universal Device Model

Joyner tiling



Array



Generic Floorplan of a binary switch

Device density

1) Upper Bound

$$n_{\max} = \frac{1}{8a^2}$$

2) IC (ITRS)

$$n_{MPU} = \frac{1}{(20a)^2}$$

How can we increase MIPS?

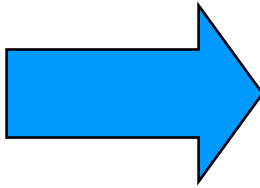
$$n = \frac{1}{8x_{\min}^2} = 6 \times 10^{12} \frac{\text{gate}}{\text{cm}^2}$$

Number of binary elements

Boltzmann-Heisenberg limit

$$\mu = k \beta^p$$

$$= \frac{n_{bit}}{t_{sw}}$$



$$\beta_{\max} \approx 10^{26} \frac{\text{bits}}{\text{s} \cdot \text{cm}^2}$$

$$t_{sw} = \frac{\hbar}{kT \ln 2} = 4 \times 10^{-14} \text{ s (300K)}$$

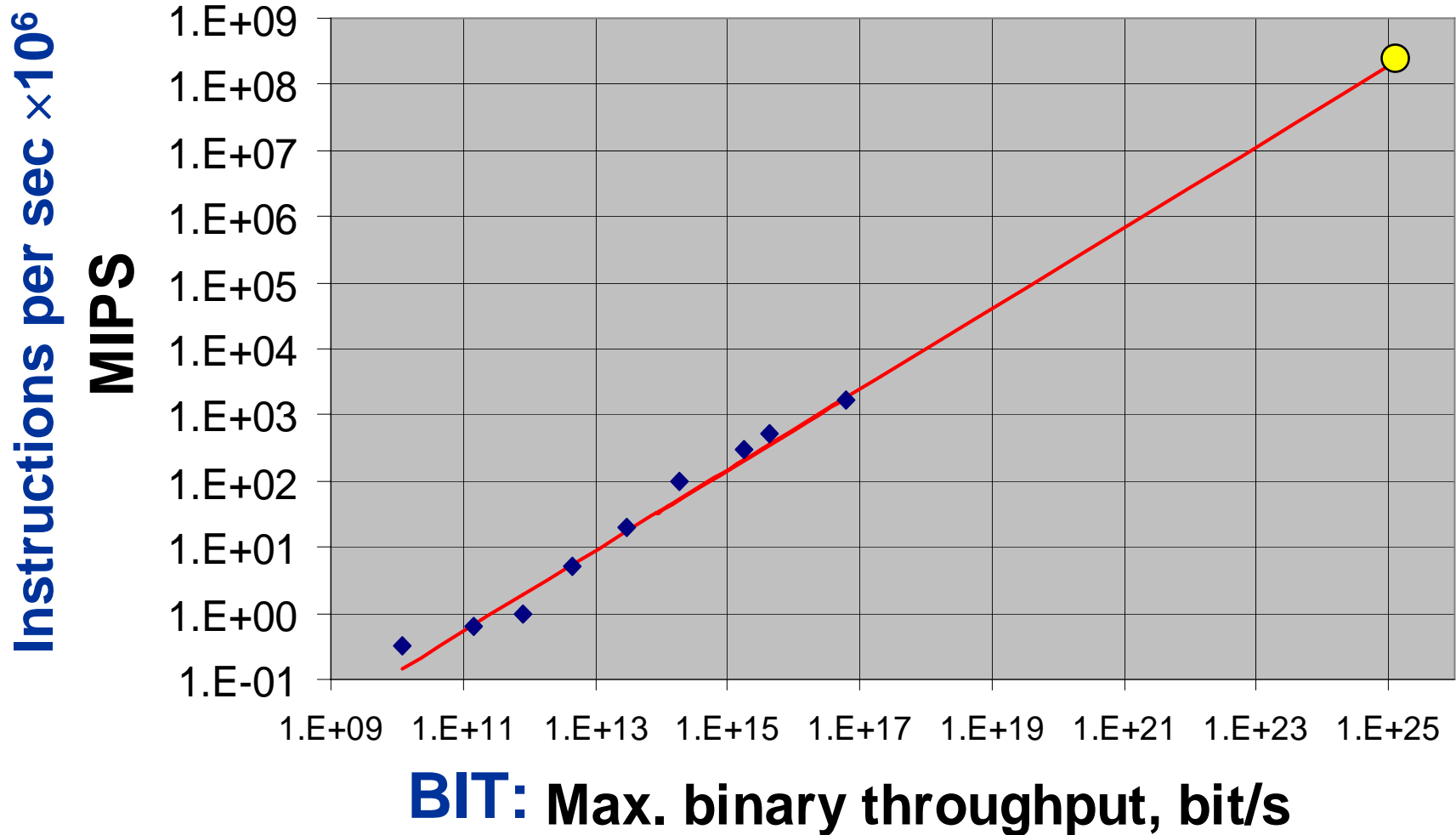
$$\mu_{\max} \approx 4 \cdot 10^8 \text{ MIPS}$$

$$k=10^{-7} \text{ and } p=0.6$$

Computing Power: MIPS (μ) vs. BIT (β)



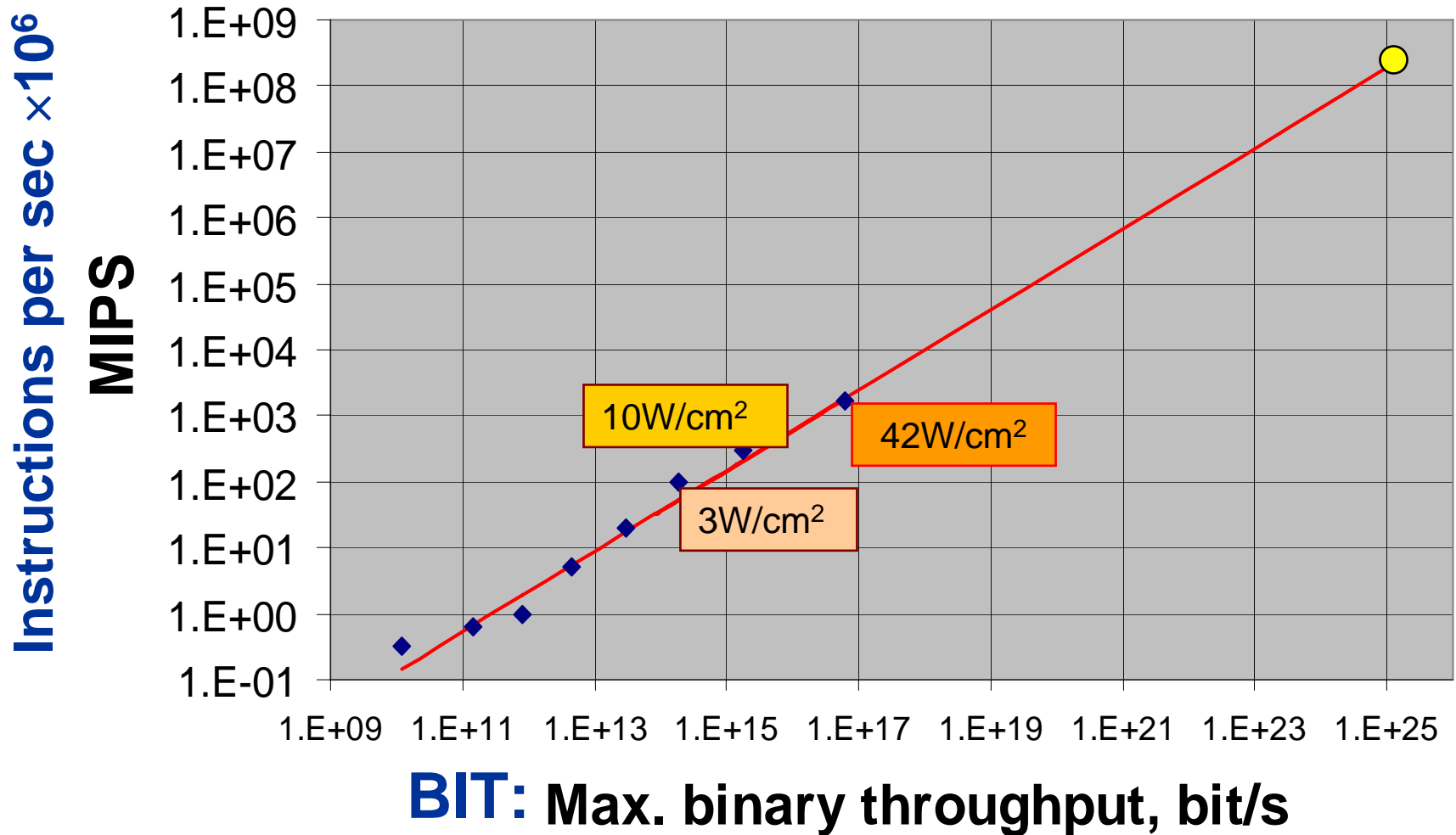
Sources: *The Intel Microprocessor Quick Reference Guide* and
TSCP Benchmark Scores



Computing Power: MIPS (μ) vs. BIT (β)



Sources: *The Intel Microprocessor Quick Reference Guide* and *TSCP Benchmark Scores*



Total Power Dissipation (@ $E_{bit} = kT \ln(2)$)

- A Catastrophe!

$$P_{chip} = \frac{n \cdot E_{bit}}{t} = 6 \cdot 10^{12} [cm^{-2}] \cdot \frac{10^{-20} [J]}{4 \cdot 10^{-14} [s]}$$

$$E_{bit} = 3k_B T \ln 2 \approx 10^{-20} J$$

$$P_{chip} = 1.5 \times 10^6 \frac{W}{cm^2}$$

The circuit would vaporize when it is turned on!

Limits of Cooling?

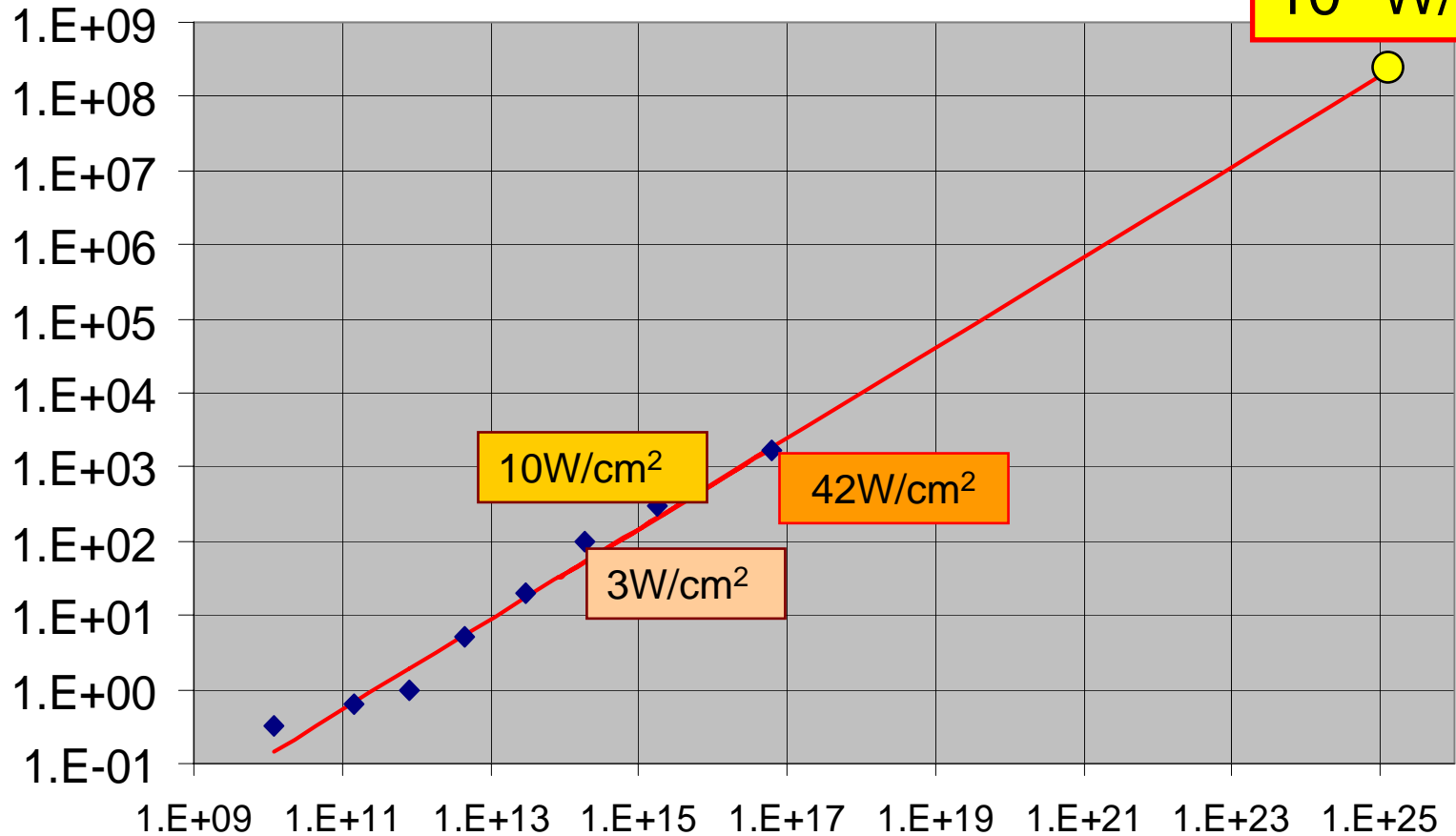
Computing Power: MIPS (μ) vs. BIT (β)



Sources: *The Intel Microprocessor Quick Reference Guide* and *TSCP Benchmark Scores*

Instructions per sec $\times 10^6$

MIPS



BIT: Max. binary throughput, bit/s

Energy Costs of Computation: Energy Consumed and Heat generated

Since each binary transition requires energy E_{bit} , the total power dissipation growth is in proportional to the information throughput:

$$P = \frac{n_{bit}}{t_{sw}} \cdot E_{bit} = \beta \cdot E_{bit}$$

BIT

$$E_b^{\min} = k_B T \ln 2$$

$$\Pi_{error} = \exp\left(\frac{E_b}{k_B T}\right)$$

MIPS

$$\mu = f(\beta)$$

Can we change f ?

We don't know how to remove that much heat!!

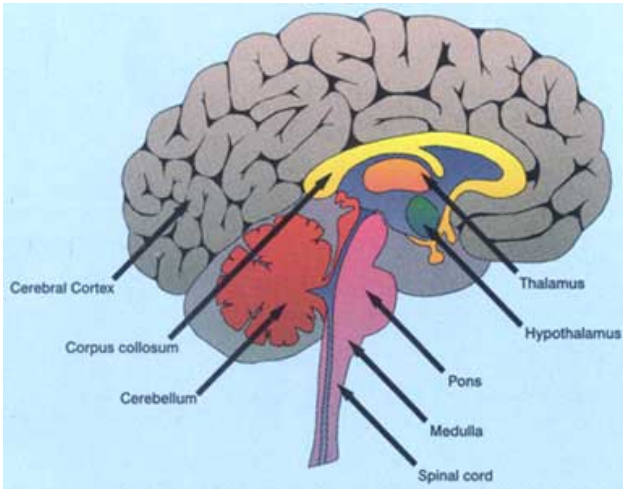
A universal relation for information processing devices

Biological Computation?

‘Computers Are Like Brains? Don’t They Wish’

The Wall Street Journal, July 9 2008

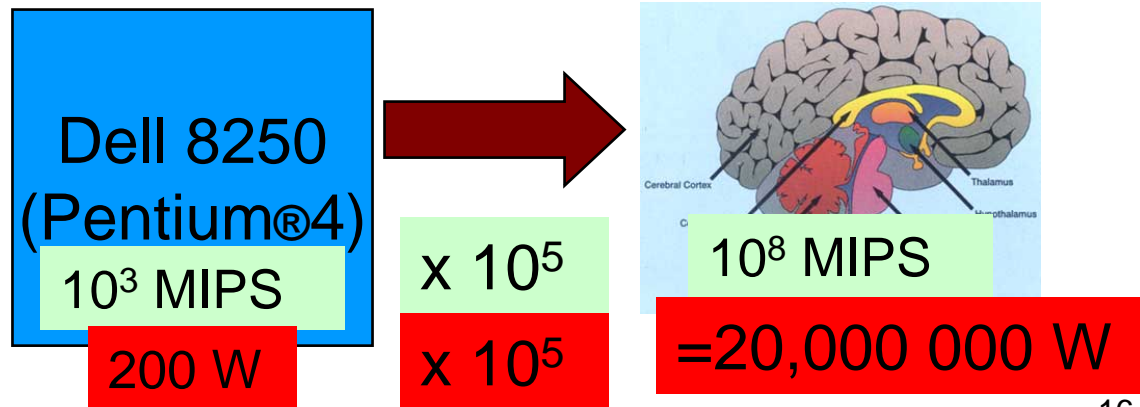
Most complex information-management system in the universe...



	Dell 8250 (Pentium® 4)	Brain
Mass	~25 kg	1.4 kg
Volume	34200 cm ³	1350 cm ³
MIPS	~10 ³ MIPS	10 ⁸ MIPS
BIT	<10 ¹⁶ bit/s	10 ¹⁹ bit/s
Power	200 W	30 W (max)
	~ 5 MIPS / W	3x10⁶ MIPS / W
	5x10⁶ k_BT / bit	700 k_BT/bit

When will computer hardware match the human brain?

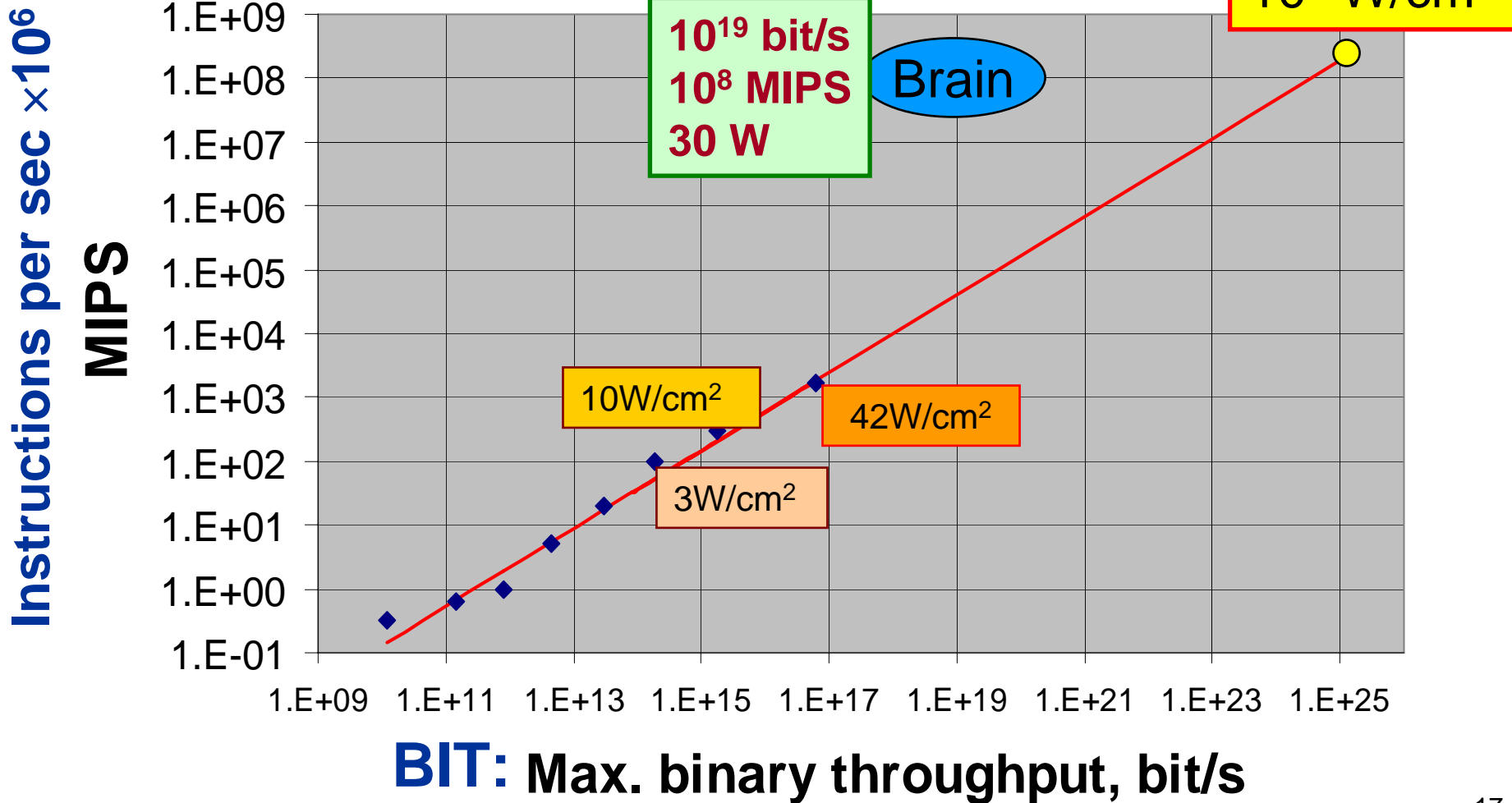
A CMOS machine at the limits of scaling would use prodigious amounts of power



Computing Power: MIPS (μ) vs. BIT (β)



Sources: *The Intel Microprocessor Quick Reference Guide* and *TSCP Benchmark Scores*



Chip Multiprocessors

Ralph K. Cavin III and Victor V. Zhirnov

Semiconductor Research Corporation

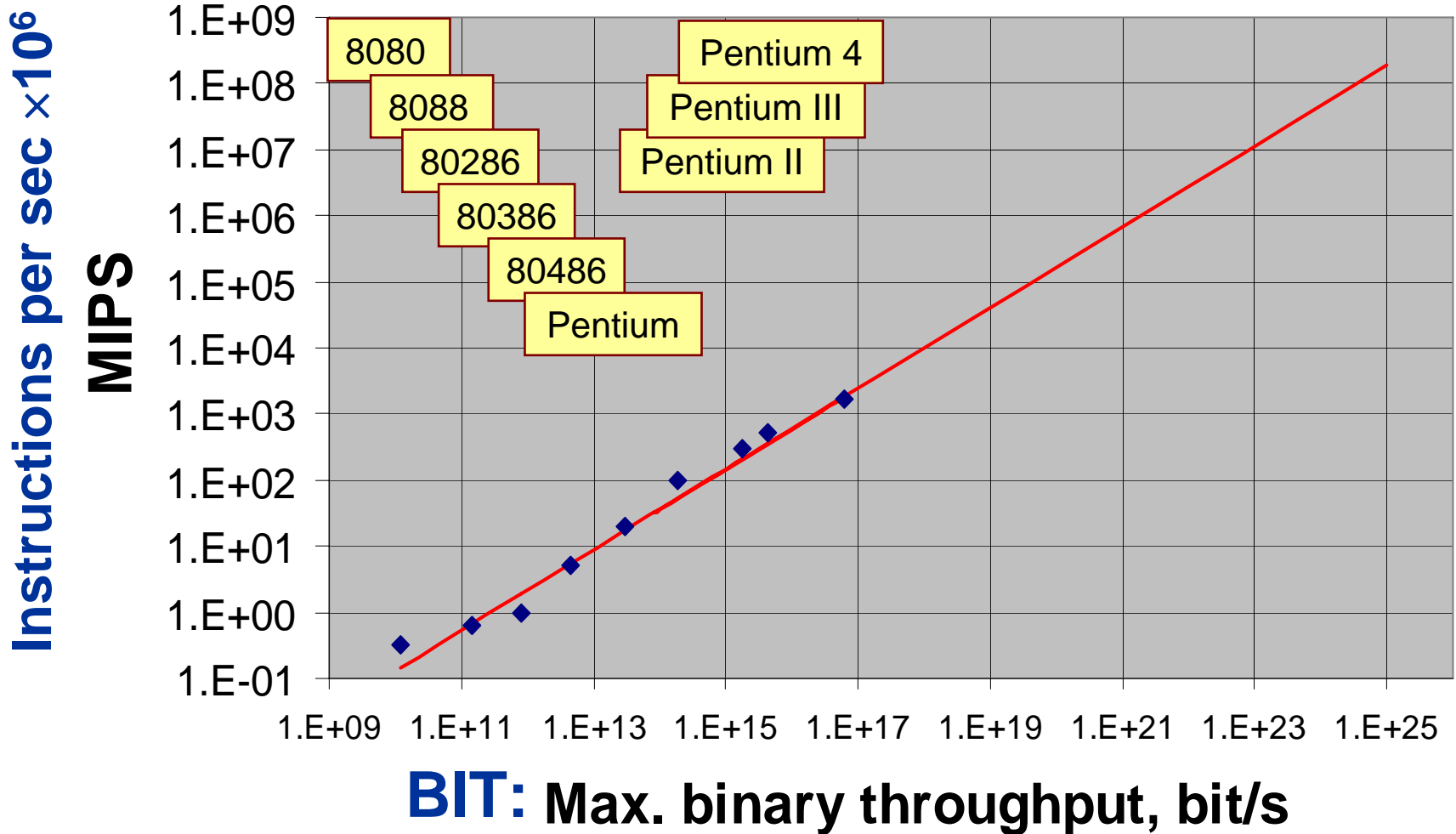
IEEE/ACM International Symposium on Nanoscale Architectures

San Jose, CA, October 21-22, 2007

Computing Power: MIPS (μ) vs. BIT (β)



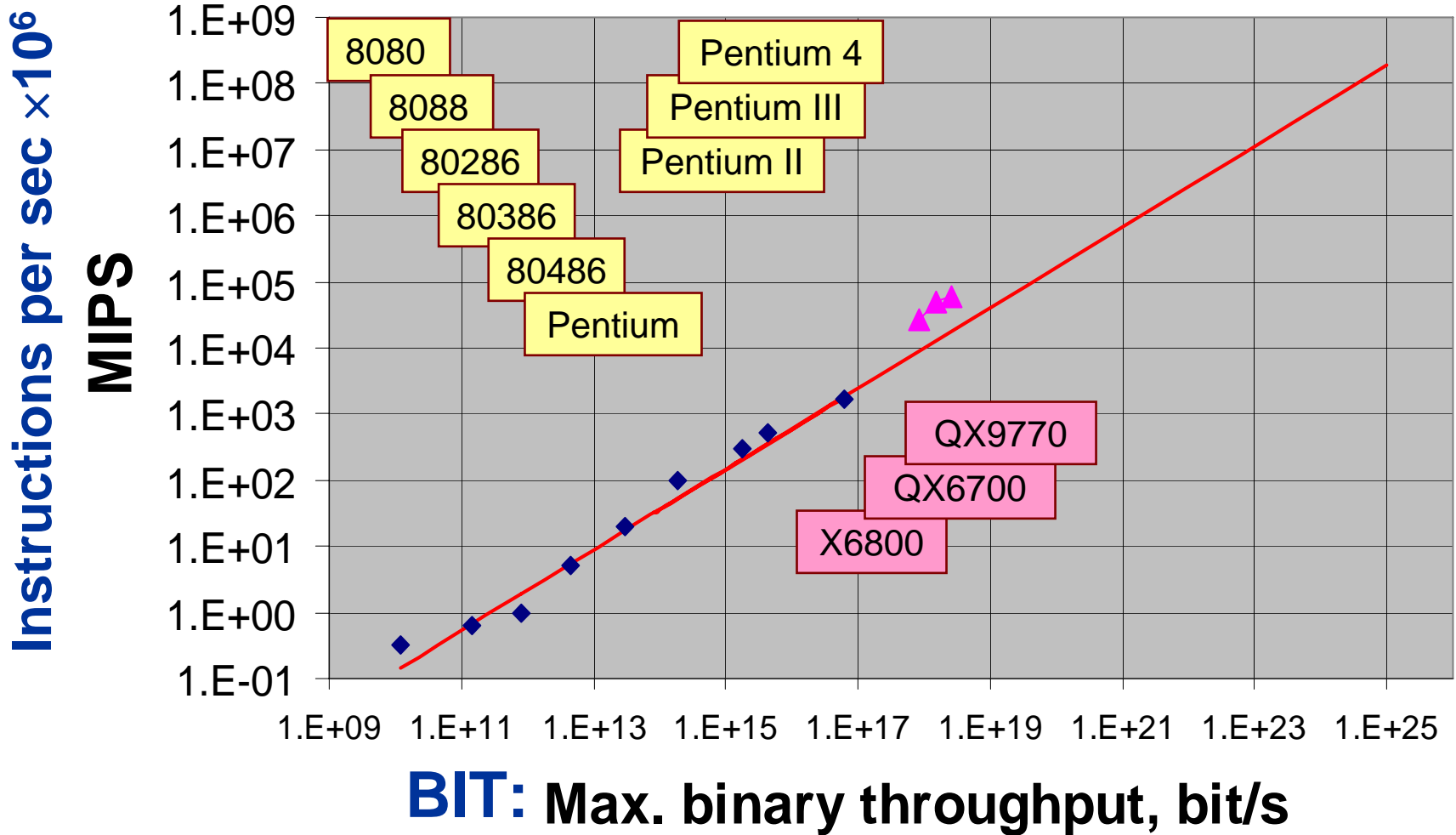
Sources: *The Intel Microprocessor Quick Reference Guide* and *TSCP Benchmark Scores*



Computing Power: MIPS (μ) vs. BIT (β)



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Multi-Core Architectures: A number K of light-weight processors instead of one heavy-weight processor

A Multi-Core processor consists of a total of N binary switches organized in K supercells or cores. Each core in this organization is a lighter-weight general-purpose information processor, containing M binary switches: $M=N/K$

$$M < N \Rightarrow E_{b_{\min}}(M) < E_{b_{\min}}(N) \quad \text{for the same error probability}$$

Favorable Multi-Core Postulates

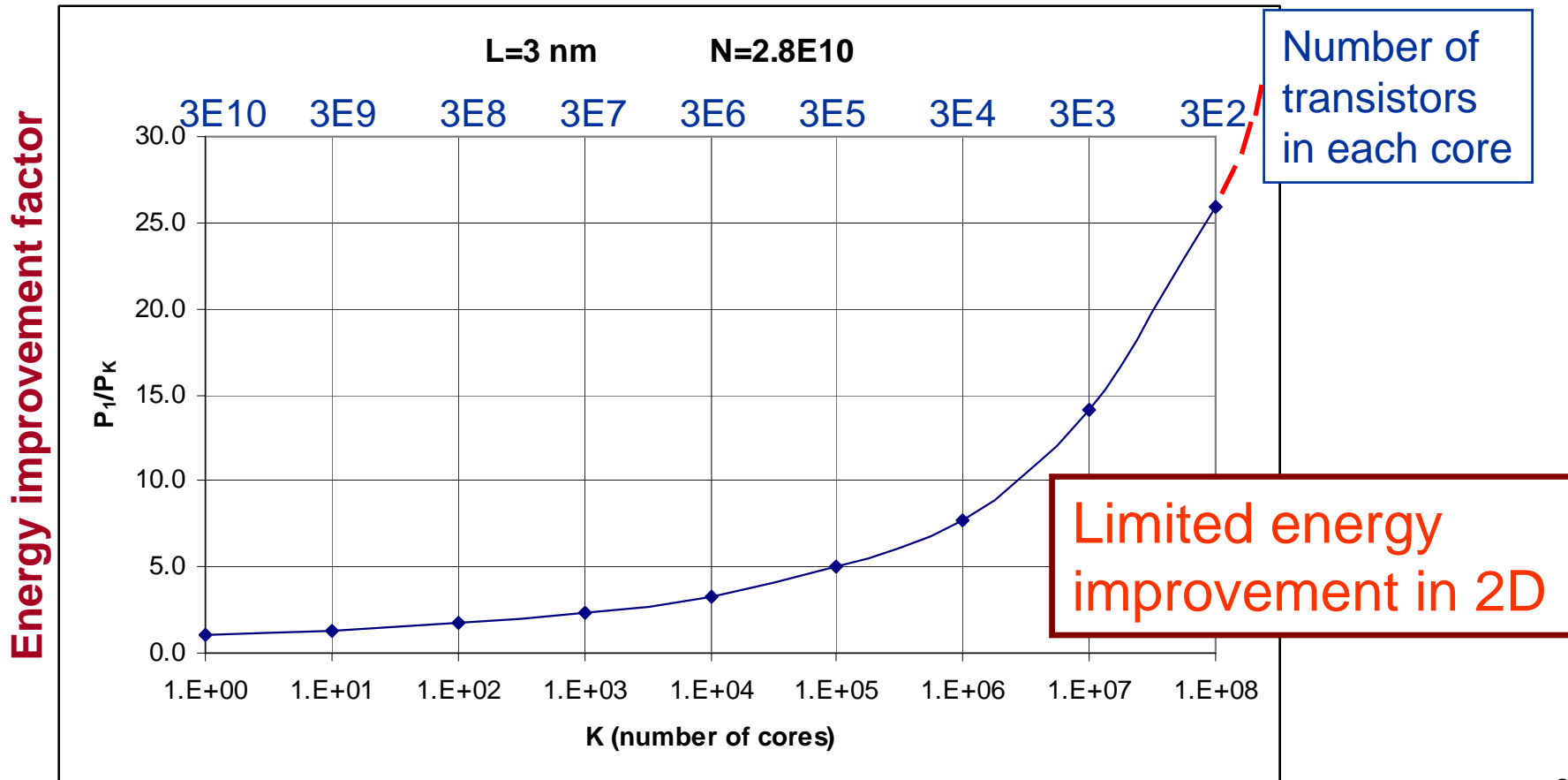
- 1) The collective action of all K cores is equivalent to the action of the single-core
- 2) All processors are engaged in useful work
- 3) Each core contains an error-detecting mechanism
- 4) The other cores are able to wait until the failed microtask computation on a core repeats the microtask to generate correct answer

Extreme Multi-Core Analysis



Power consumption
by K cores:

$$P_K = K \cdot \frac{M}{t_{sw}(M)} \cdot E_{b_{\min}}(M) = \frac{N}{t_{sw}(M)} \cdot E_{b_{\min}}(N)$$



“Coreness” / “Weight”- Dilemma



- ◆ There is a limit for a maximum number of transistors in 1 cm² of chip area

$$N_{\max} \sim 10^{10} \text{ cm}^{-2} \quad (L_g=5 \text{ nm})$$

- ◆ A Multi-Core Information processor consists of a total of N binary switches organized in K supercells or cores.
 - ❖ Each core in this organization is a lighter-weight general-purpose information processor, containing M binary switches: $M=N/K$

- ❖ In the limit:
$$M = \frac{10^{10}}{K}$$

What is smallest M ?

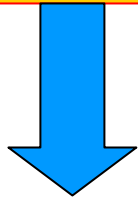
System scaling limits need to be understood

Different Facets of Scaling



Device Scaling

Decrease the physical size

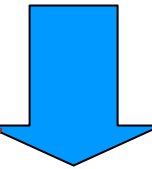


Increased Functionality

Increase system capability and/or application space

System Scaling

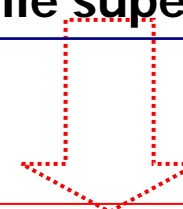
Decrease physical size of the system and increase both system capability and application space



Example:

Ultra Mobile Platform

'mobile supercomputer'

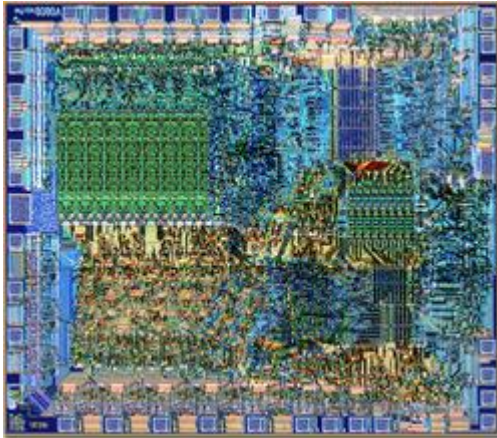


Extreme Microsystems

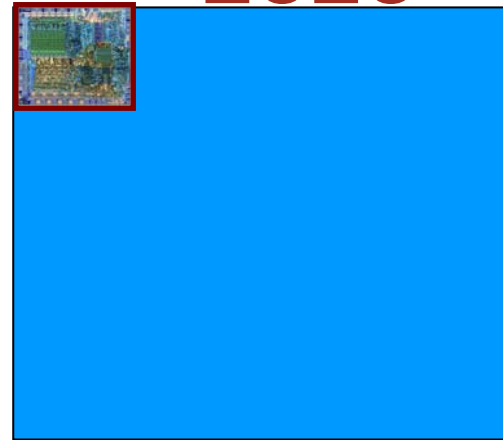
Electronic cell

Scaling of 8080 MPU

1974



2020



Technology:	NMOS
Feature size:	6 μm
# of transistors:	4500
Die size:	5 mm x 4 mm
Voltage:	5V, 12 V
Frequency:	2 MHz
Power:	1.5 W

Technology:	CMOS
Feature size:	6 nm
# of transistors:	4500
Die size:	5 μm x 4 μm
Voltage:	~0.5 V
Frequency:	~2 MHz-1GHz
Power:	~10nW-10 μW



◆ Multi-core CPU

- ❖ What is the maximum possible number of cores in multi-core processors

◆ ‘Mobile supercomputers’

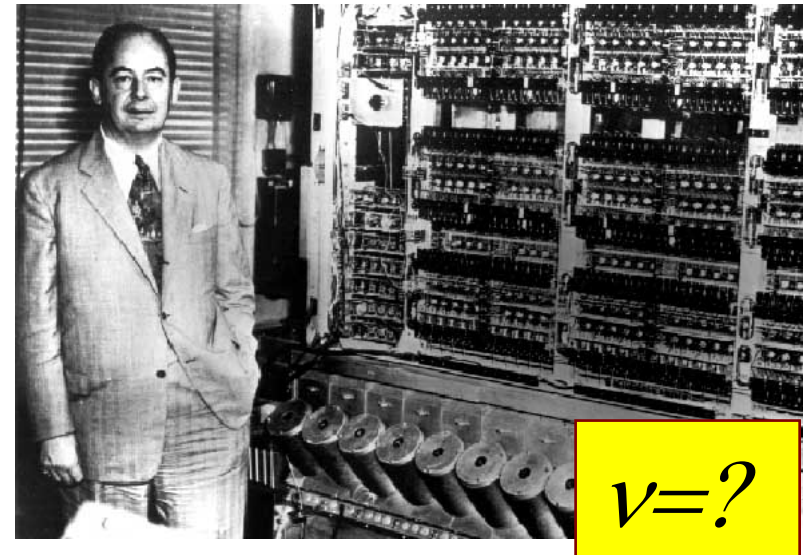
- ❖ What is the smallest possible size of an intelligent ‘piconode’?

A large cyan arrow pointing to the right, containing the text 'Minimal Turing Machine'. A red bracket on the left side of the slide groups the two bullet points under this arrow.

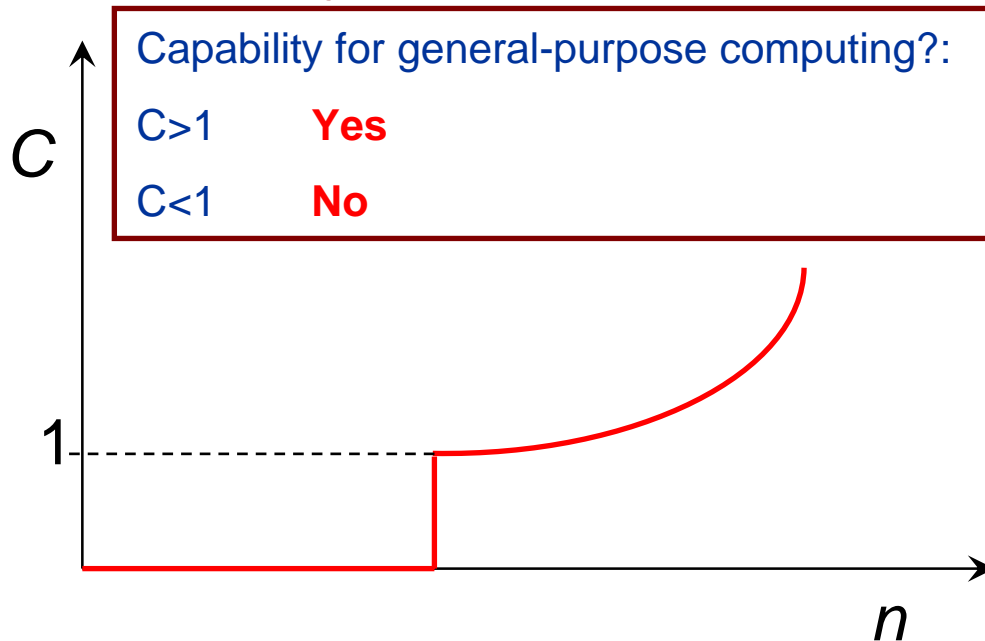
Minimal Turing
Machine

Von Neumann's Threshold

"If one constructs the automaton (A) correctly, then any additional requirements about the automaton can be handled by sufficiently elaborated instructions. This is only true if A is sufficiently complicated, if it has reached a certain minimum of complexity" (J. von Neumann)



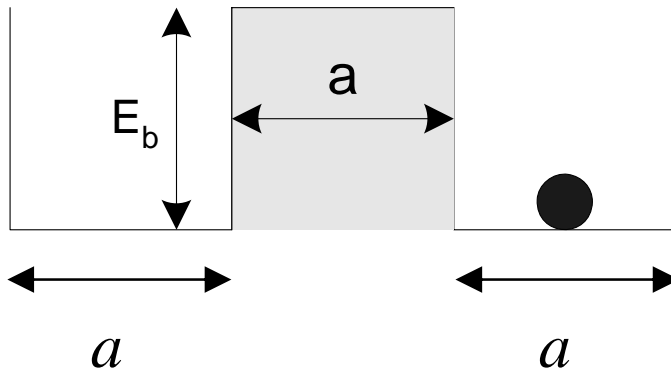
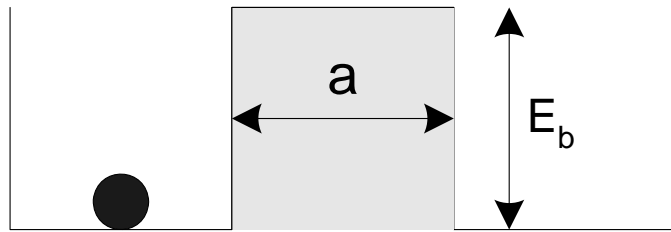
$v=?$



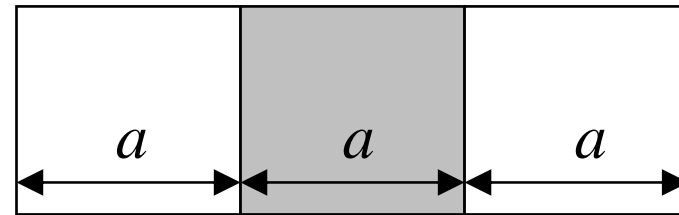
Von Neumann threshold

'Minimal' Turing Machine

Binary switch abstraction: Generic floorplan and energetics



Generic Floorplan of a binary switch



$$a = \frac{\hbar}{\sqrt{2mkT \ln 2}} = 1.5 \text{ nm}$$

$$Area_{\min} = 3a^2$$

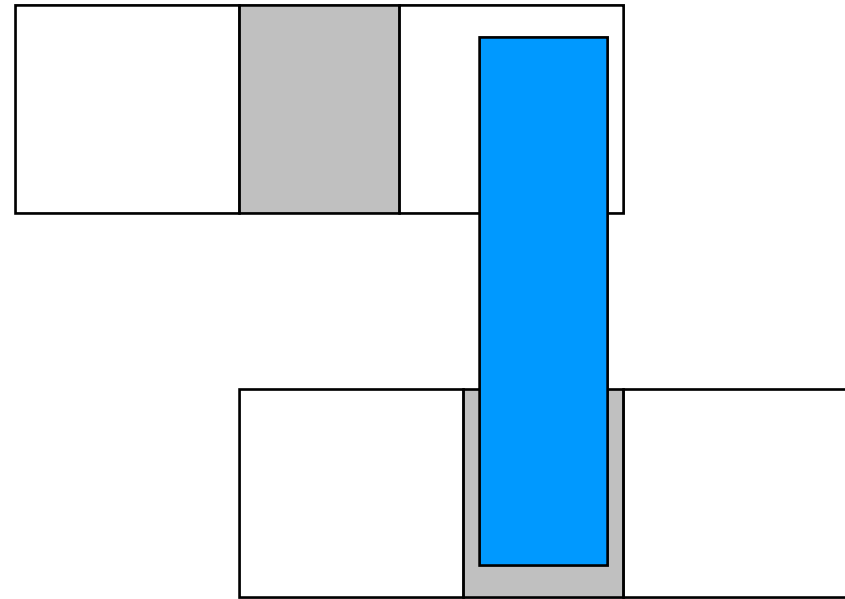
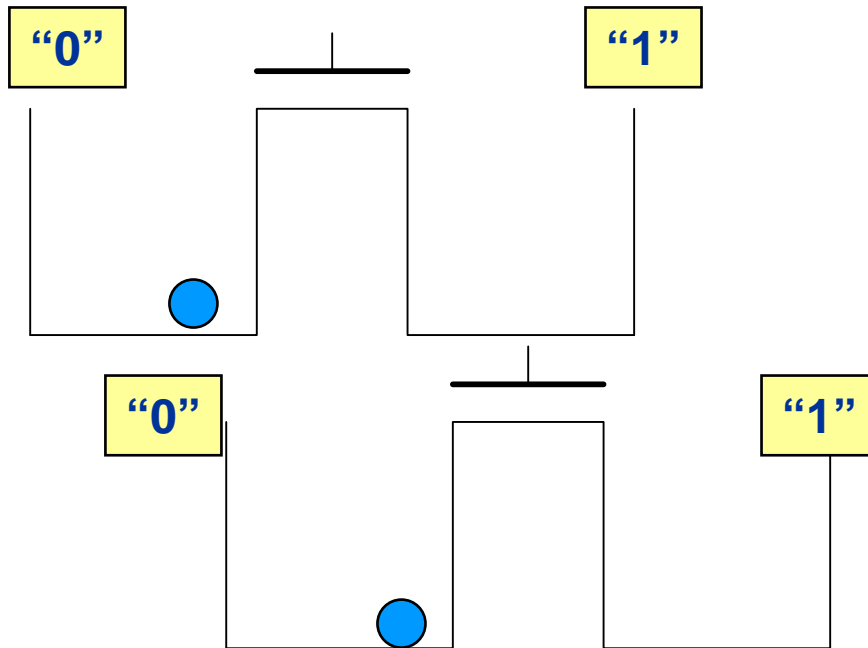
$$E_{sw_{\min}} = 3k_B T$$

$$\varepsilon = k_B T \left(\frac{J}{\text{tile}} \right)$$

Connected Binary Switches



Functional View

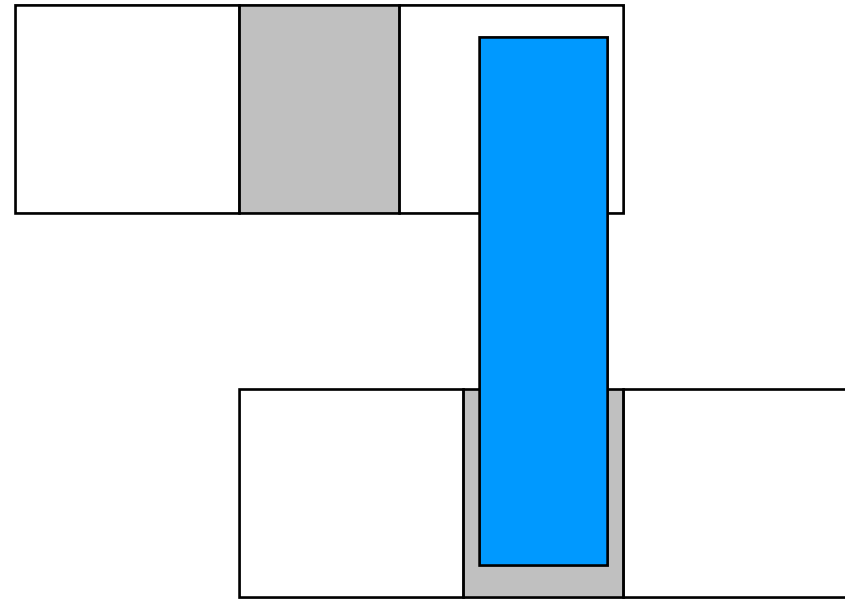
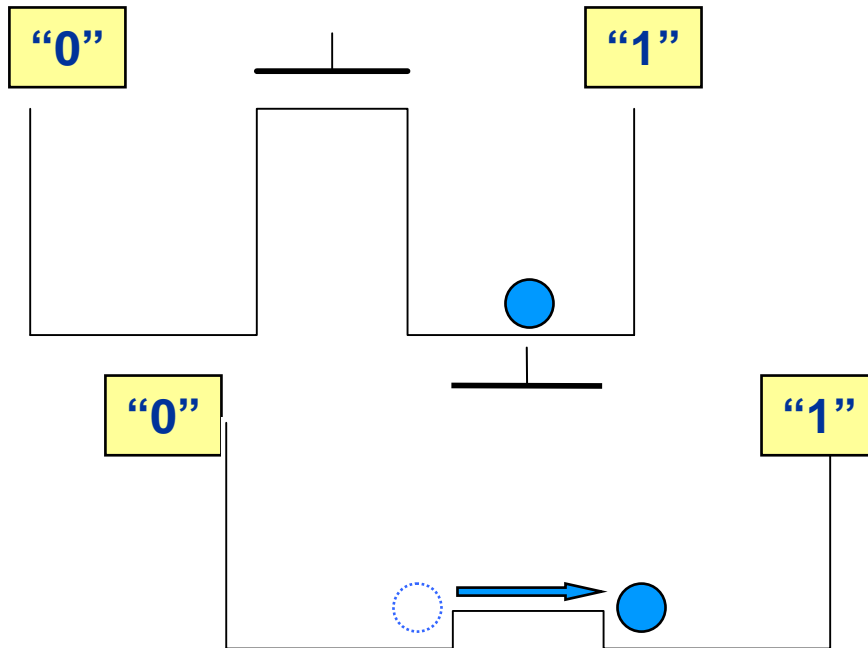


Physical View

Information-bearing charge

Connected Binary Switches

Functional View



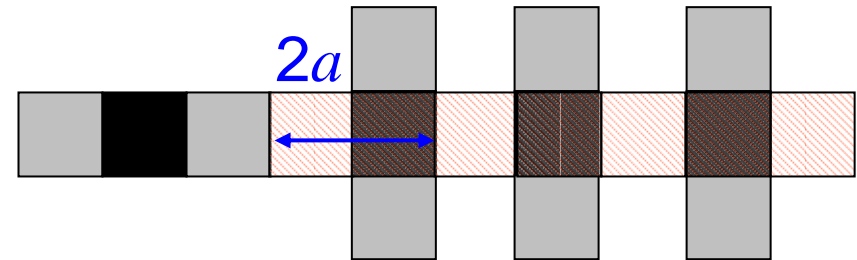
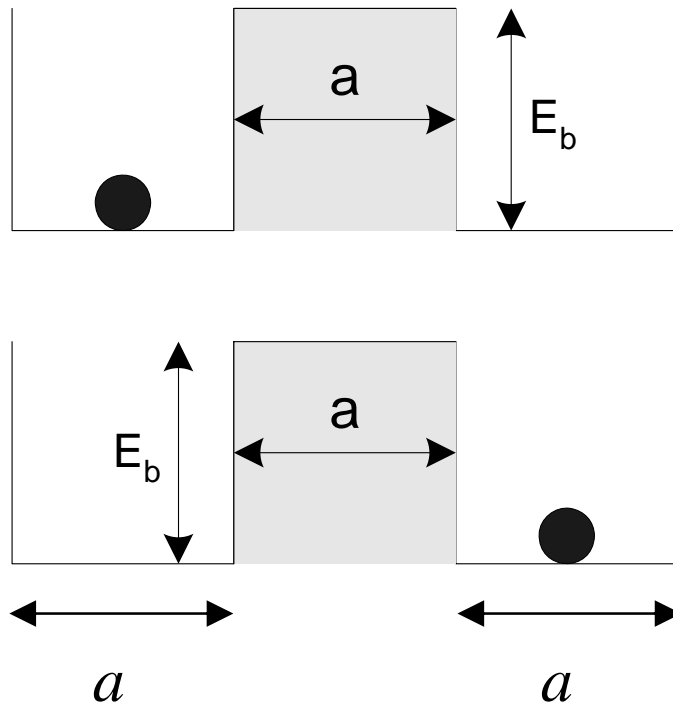
Physical View

Information-bearing charge

=

Barrier-forming charge

Interconnect abstraction: *Extended Well Model*



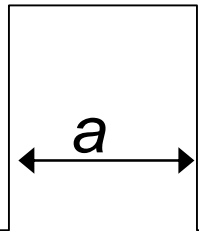
$$L_{\min} = 2a \cdot F$$

Connecting Binary Switches via Wires:

Extended Well Model



The problem is to 'place' the electron on the down stream gate – more than one electron is needed to 'charge' the line

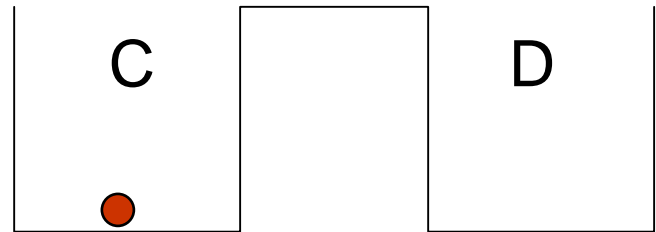
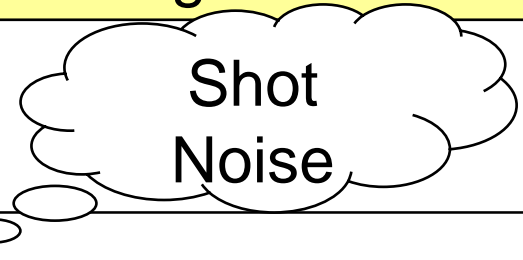


A

B

$$\Pi_{CD} = \frac{a}{L}$$

L



Example: $L=4a$

$N=1 \rightarrow \Pi < 0.25$

In General:

$$\Pi = 1 - \left(1 - \frac{a}{L}\right)^N$$

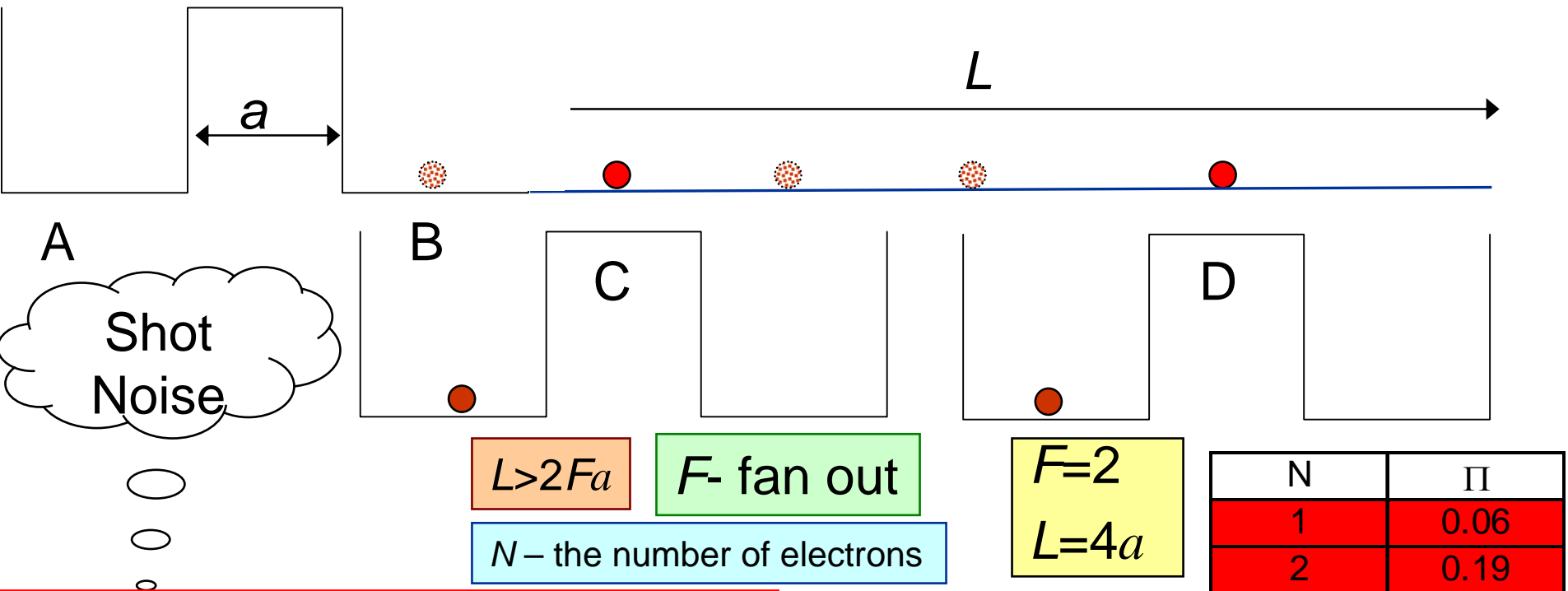
N – the number of electrons

Connecting Binary Switches via Wires

($L > 4a$, N electrons)



For logic operation, a binary switch needs to control at least two other binary switches



$$\Pi_{C\&D} = \Pi_C \times \Pi_D = \left(1 - \left(1 - \frac{a}{L} \right)^N \right)^2$$

N	Π
1	0.06
2	0.19
3	0.33
4	0.47
5	0.58
6	0.68

Minimum number of electrons in interconnect line for communication and fan-out

N - number of electrons

F - fan-out

k - number of tiles

$$\Pi = \left(1 - \left(1 - \frac{a}{L} \right)^N \right)^F$$

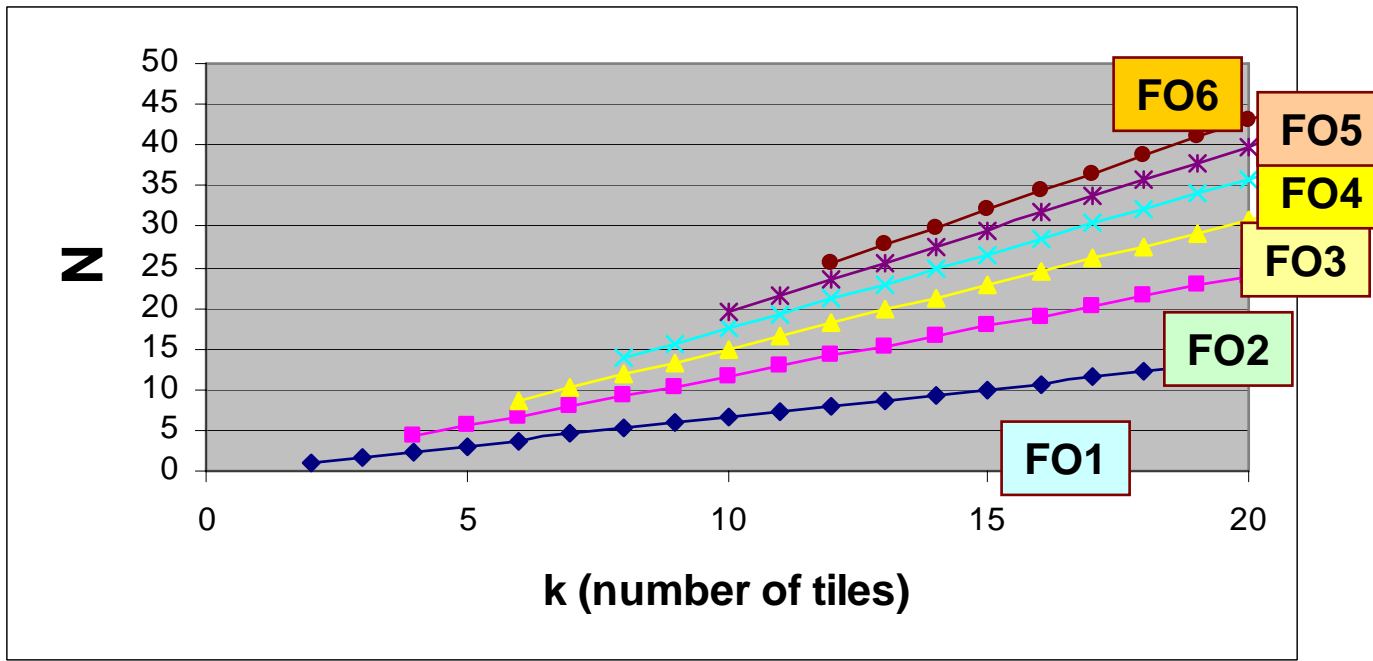
$\frac{L}{a} = k$

→

$$\frac{1}{2} = \left(1 - \left(1 - \frac{1}{k} \right)^N \right)^F$$

$\Pi = \frac{1}{2}$

$$N = \frac{\ln \left(1 - \frac{1}{2^F} \right)}{\ln \left(1 - \frac{1}{k} \right)}$$



Minimum switching energy for connected binary switches



$$E_{sw} = 3E_b + NE_w = (N+3)k_B T \ln 2$$

Minimum fan out

$$F=2 \quad L=4a$$

$$N_{min} = 5$$

$$E_{sw} = 8k_B T \ln 2$$

Typical fan out

$$F=4 \quad L=8a$$

$$N_{min} = 14$$

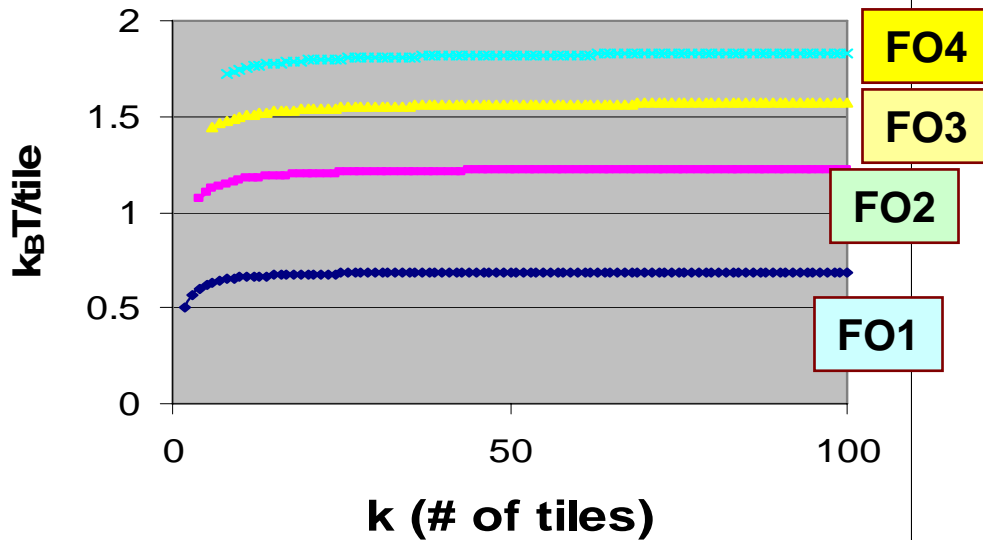
$$E_{sw} = 17k_B T \ln 2$$

Communication between binary switches takes more energy than does changing switch state

Can we make communication more energy efficient?

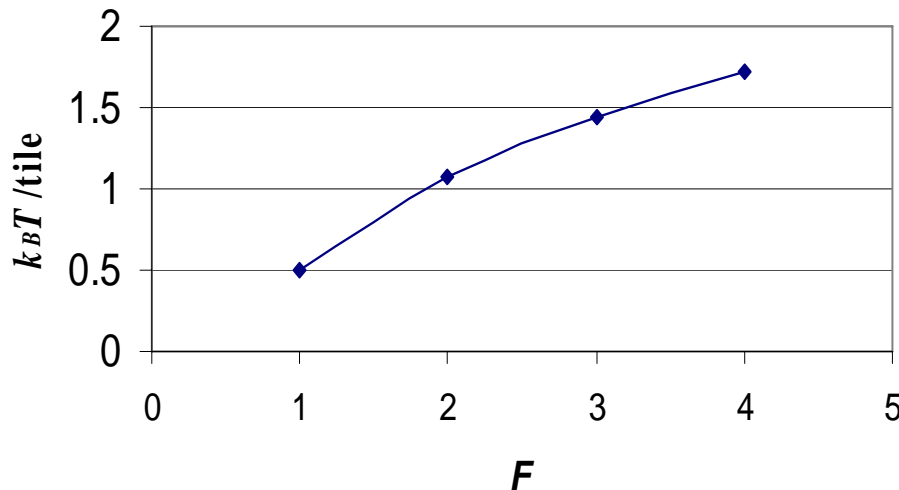
N	Π
1	0.00
2	0.00
3	0.01
4	0.03
5	0.06
6	0.09
7	0.14
8	0.19
9	0.24
10	0.29
11	0.35
12	0.41
13	0.46
14	0.51
15	0.56
16	0.60
17	0.65
18	0.68

Energy per interconnect tile



Long interconnect limit

$$\langle \varepsilon \rangle = 1.33 \frac{k_B T}{\text{tile}}$$



Minimum interconnect limit

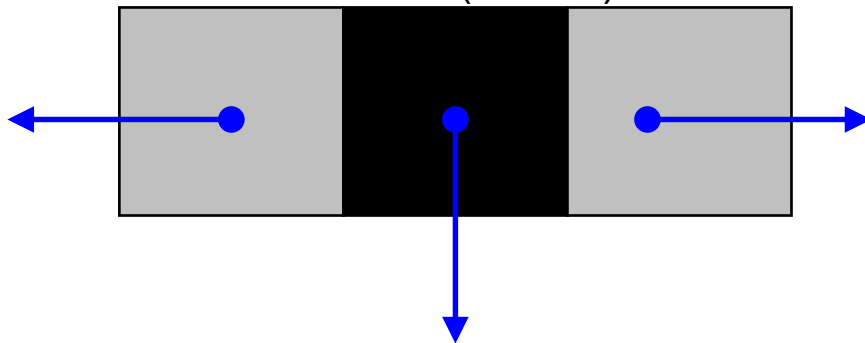
$$\langle \varepsilon \rangle = 1.18 \frac{k_B T}{\text{tile}}$$

$$\varepsilon \sim k_B T / \text{tile}$$

Floorspace Expenses of Communication between Binary Switches

Assumption: For each of 3 tiles of Binary Switch and for a fan-out of **three**, we need at least:

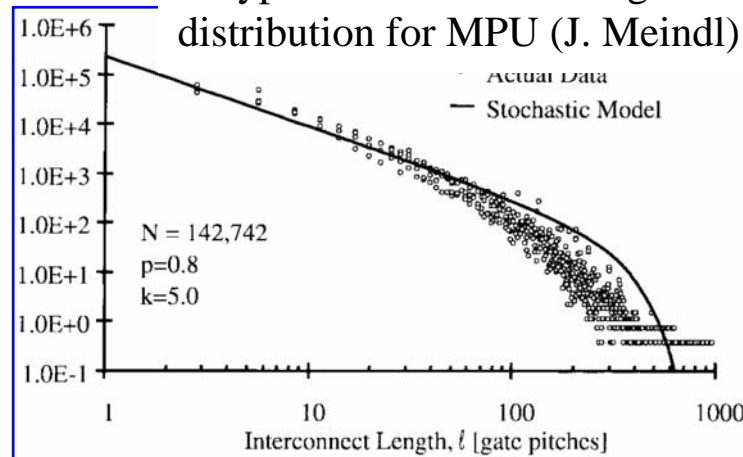
One contacting interconnect tile (3 total) and one connecting interconnect tile (3 total)



Total **6 interconnect** tiles per binary switch

$$L_{\text{int}} \sim 6a$$

A typical interconnect length distribution for MPU (J. Meindl)



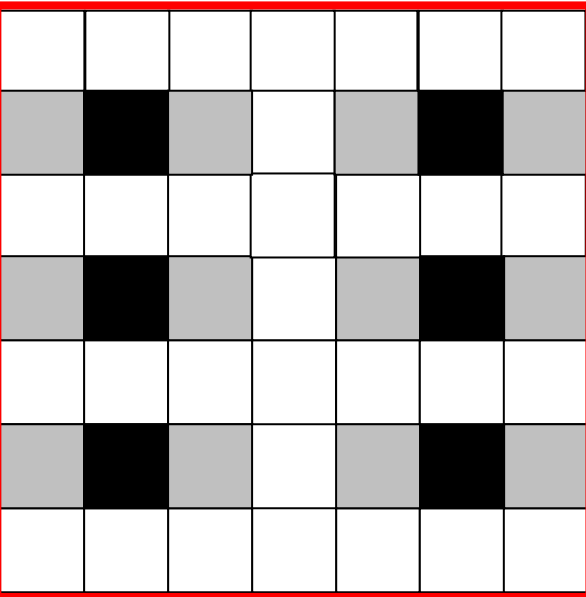
n, cm^{-2}	$\bar{L}(n)/L_g$
1.E+02	4.1
1.E+04	6.4
1.E+06	8.3
1.E+08	9.7
1.E+10	10.5

Reality check:

Digital circuit abstraction: Generic floorplan and energetics and speed



Switching energy of one binary switch in a circuit (FO3)



3 switch tiles

$$E_{sw} = 3E_b + 6E_b = 9k_B T \ln 2$$

6 wire tiles

Operational energy of a circuit of
 n binary switches:

(50% activity)

$$E_{op} = \frac{9}{2} n k_B T \ln 2$$

$$Area_{min} = n \cdot 8a^2$$

Joyner tiling

Switching delay of one binary switch in a circuit:

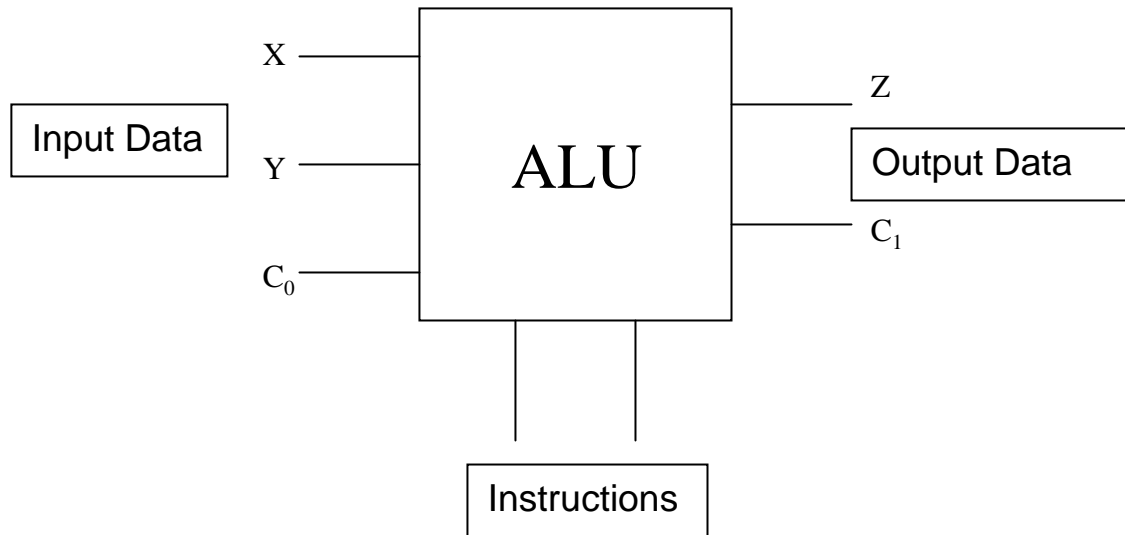
Speed: $\tau_{min}/tile$

$$\tau_{min} = \frac{\hbar}{kT \ln 2}$$

~40 fs

$$t_{sw} = 9 \tau_{min}$$

1-bit ALU



The minimal ALU does $2^2=4$ operations on two 1-bit **X** and **Y**:

Operation 1: **X AND Y**

Operation 2: **X OR Y**

Operation 3: **(X+Y)**

Operation 4: **(X+(NOT Y))**



Jan Rabaey,
Digital Integrated Circuits

Minimal ALU abstraction: *Energetics*



$$E_{ALU} = \frac{9}{2} \cdot 98 \cdot k_B T \ln 2 \sim 300 k_B T$$

Energy efficiency: $\eta = \frac{E_{op}}{E_{ALU}}$

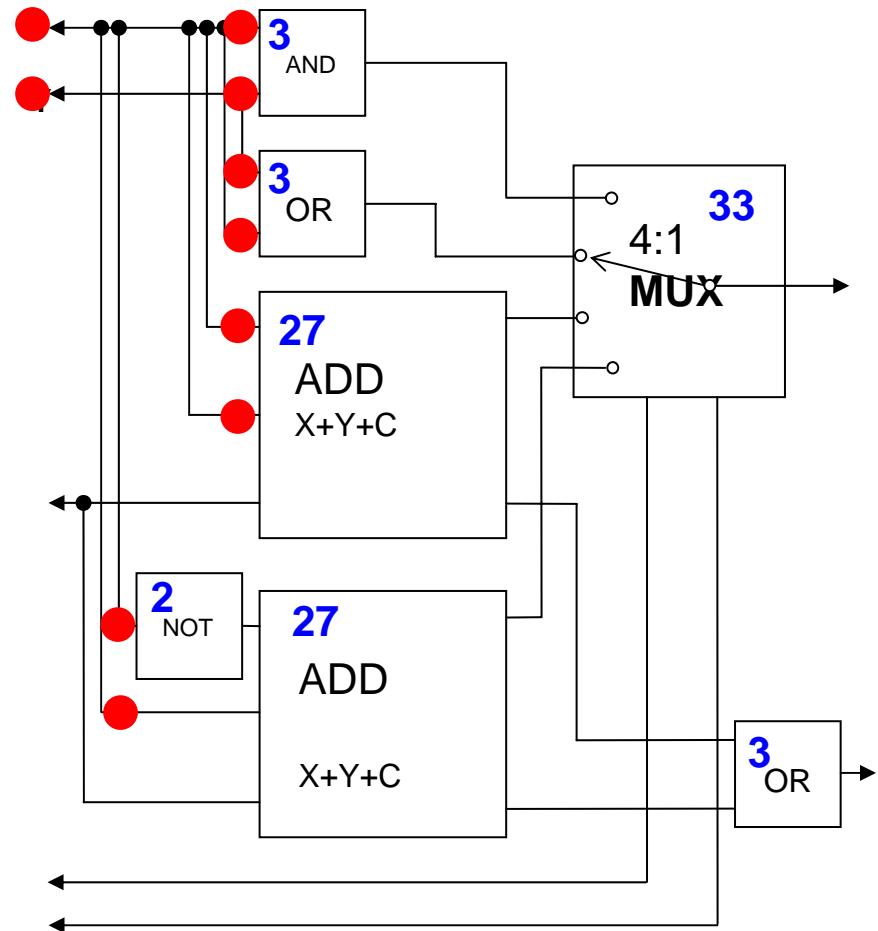
$$E_{AND} = \frac{9}{2} \cdot 3 \cdot k_B T \ln 2 \sim 9 k_B T$$

$$\eta_{AND} \sim 3\%$$

$$E_{ADD} = \frac{9}{2} \cdot 27 \cdot k_B T \ln 2 \sim 84 k_B T$$

$$\eta_{ADD} \sim 28\%$$

All 4 units execute even though only one output is used



Total: 98 devices

Can we increase ALU efficiency?



◆ De-parallelize inputs ?

- ❖ Two input selectors are needed
 - Two 1:4 DMUX
 - 33 devices each

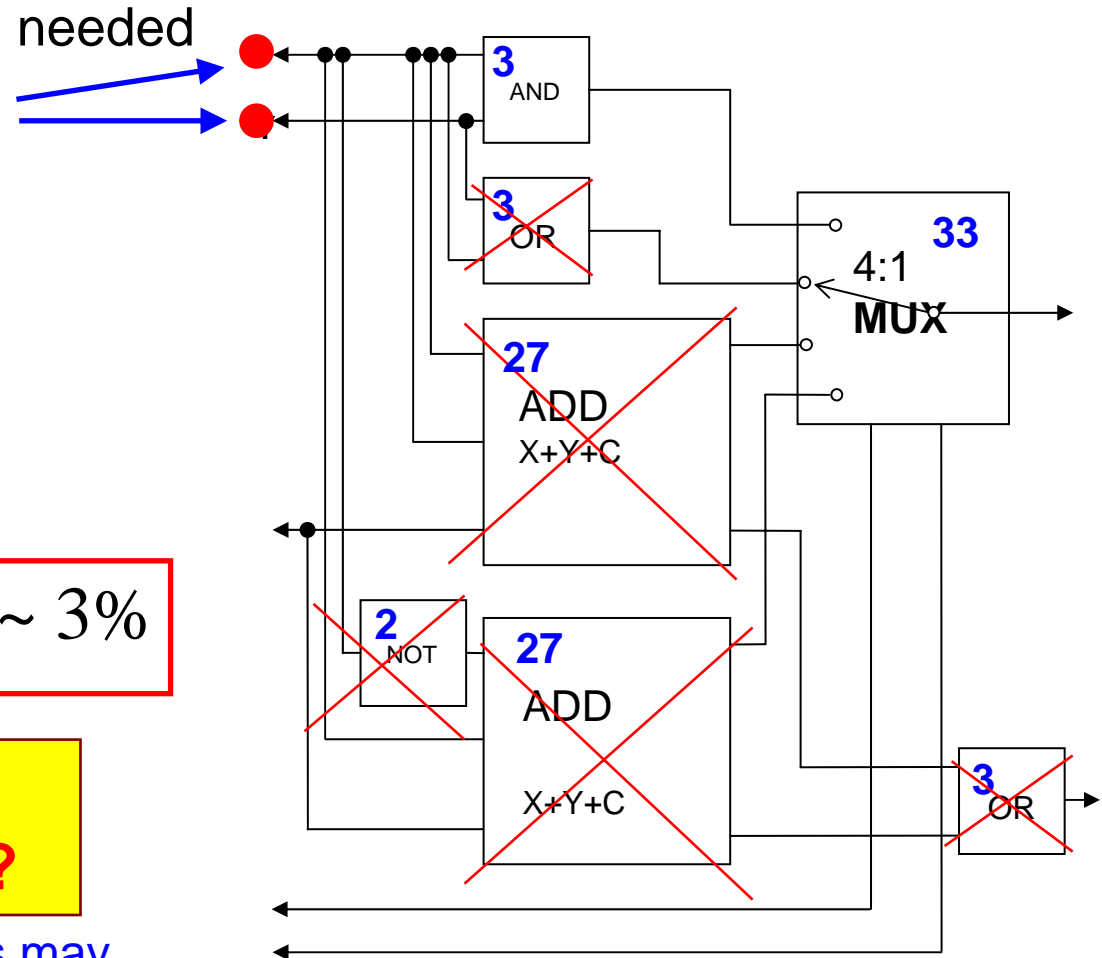
Example: AND operation

Active device count: 101

$$\eta_{AND} \sim 3\%$$

Carnot's equivalent for Computational Engine?

Thermodynamic entropy analysis may provide new insight on chip design



Minimal ALU abstraction: *Timing*



$$t_1 = 9\tau_{min}$$

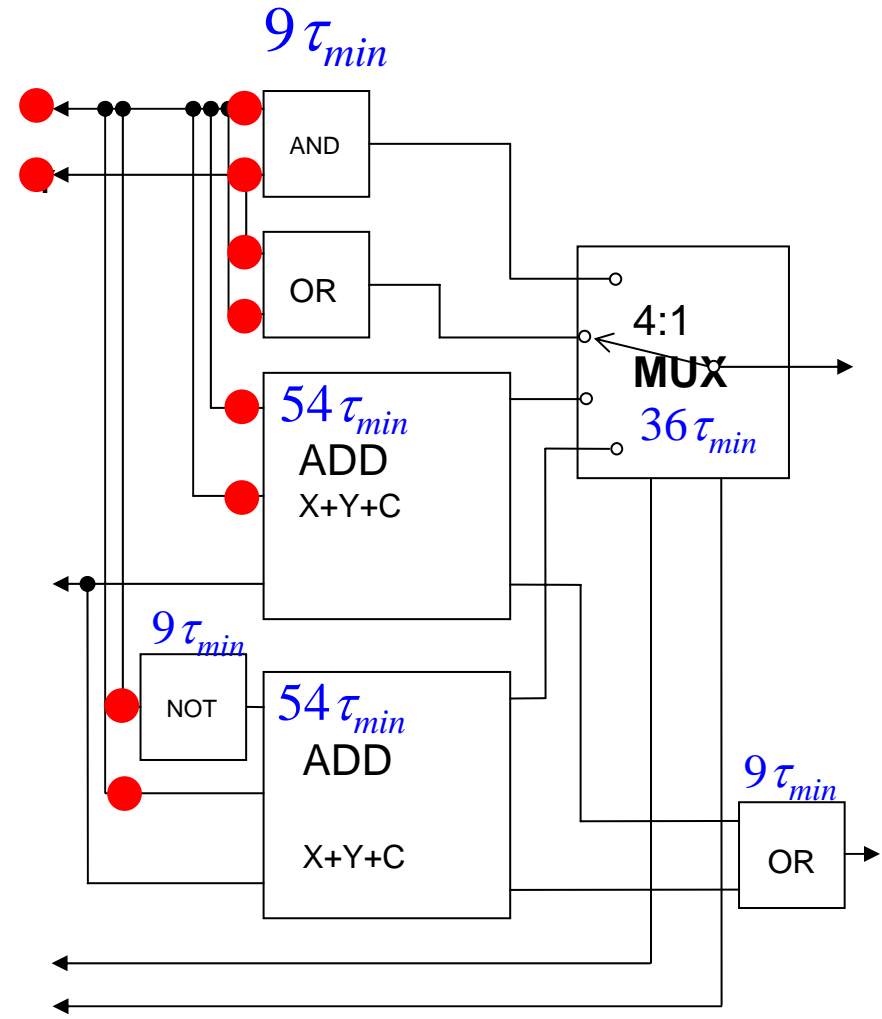
~360 fs

$$t_n = n \times 9\tau_{min}$$

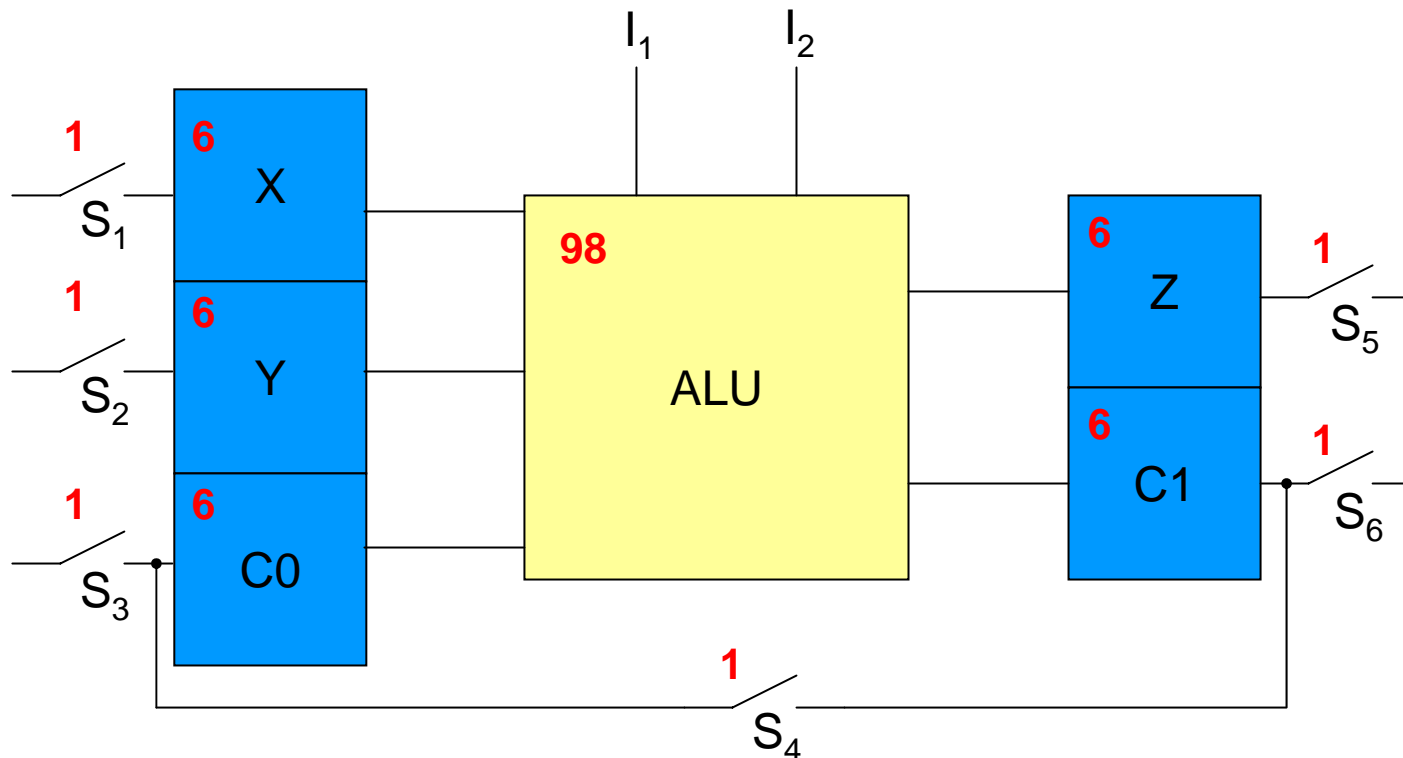
n = # cascades

$$t_{ALU} \sim 50\tau_{min}$$

~2 ps

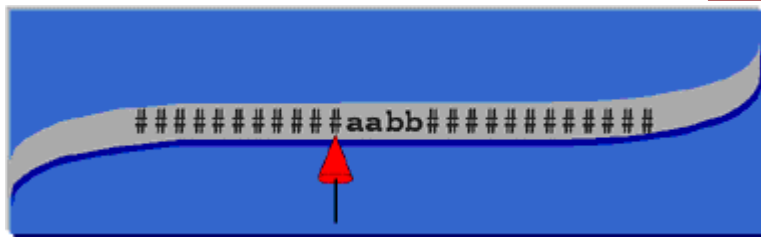


Minimal CPU



Total: 134 devices

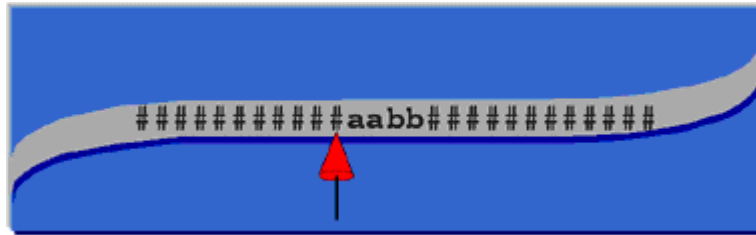
$$E_{CPU} = \frac{9}{2} \cdot 135 \cdot k_B T \ln 2 \sim 420 k_B T / cycle$$



Not included

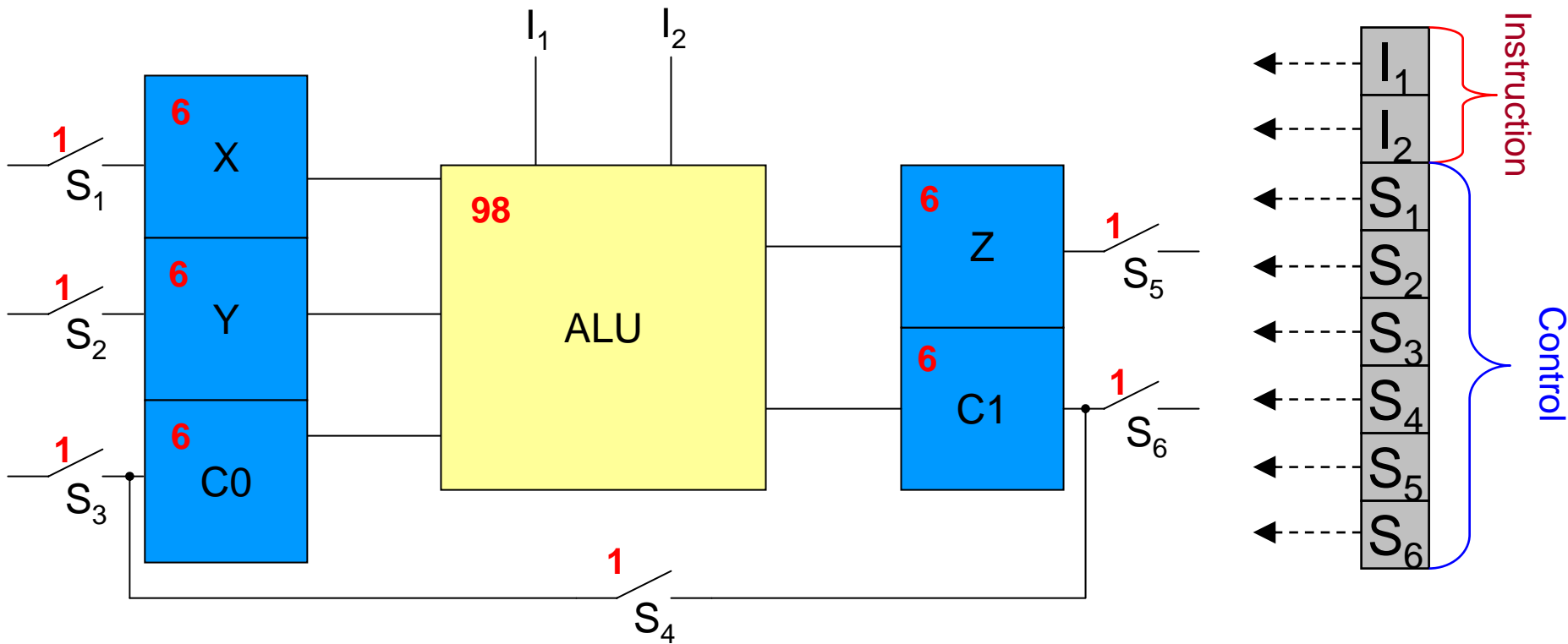


Minimal Turing Machine



Memory

Program Counter



8 bit per cycle

Program Memory per operation

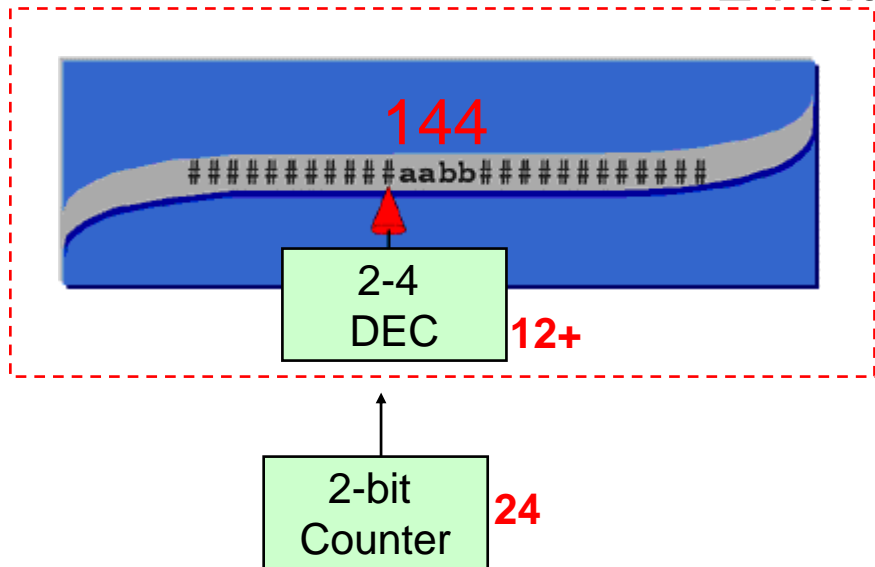


- Operation 1: **X AND Y**
- Operation 2: **X OR Y**
- Operation 3: **(X+Y)**
- Operation 4: **(X+(NOT Y))**

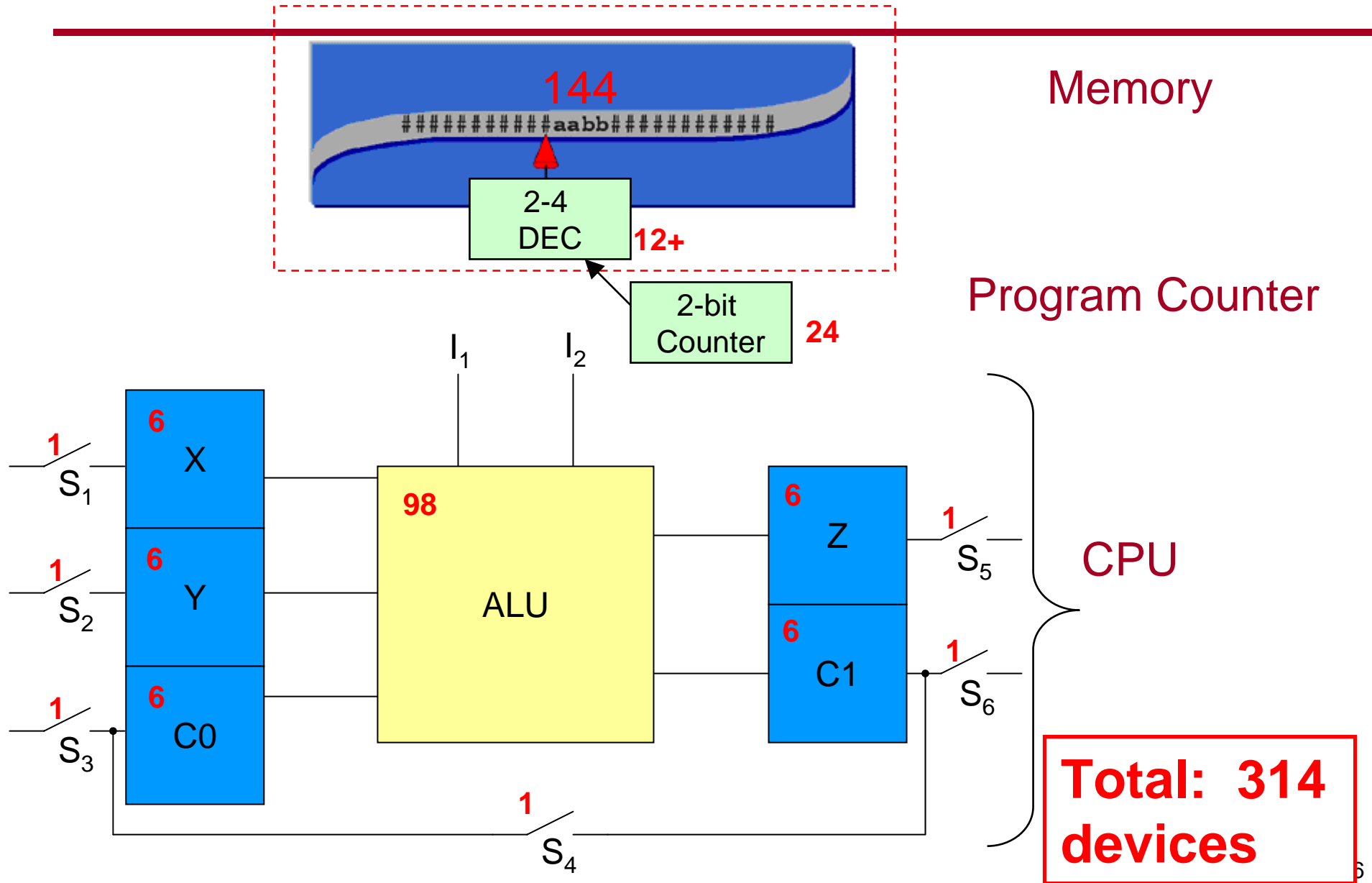
3 cycles per ALU operation

IN
Op
OUT
8 bit per cycle

24 bit Memory per operation



Minimal Turing Machine



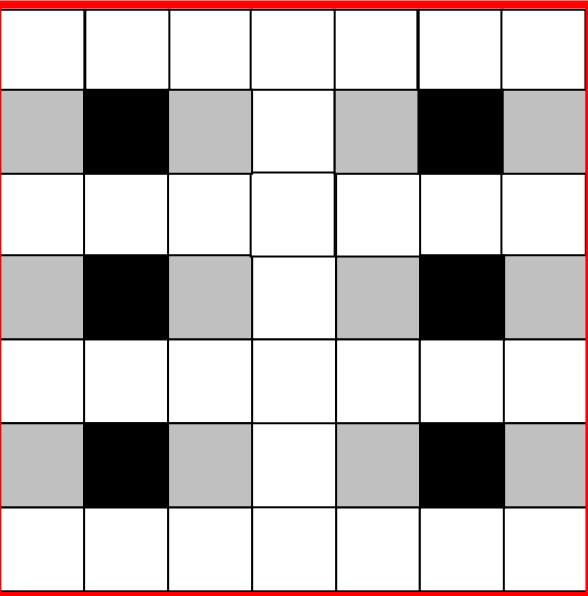
Turing Machine Implementation: *Generic floorplan and energetics*

Von Neumann threshold:

$$n=314$$

Joyner tiling:

$$Area_{\min} = n \cdot 8a^2 = 314 \cdot 8a^2 \approx 2500a^2 = 50a \times 50a$$



$$a_{\min} = 1.5 \text{ nm}$$

$$Area_{\min} = 75 \text{ nm} \times 75 \text{ nm}$$

Operational energy of the
Minimal Turing Machine

$$E_{op} = \frac{9}{2} nk_B T \ln 2 \approx 980 k_B T / \text{cycle} = 4 \cdot 10^{-18} \frac{\text{J}}{\text{cycle}}$$

Per full CPU operation:

$$E_{op} = 3 \cdot 4 \cdot 10^{-18} \frac{\text{J}}{\text{cycle}} \approx 10^{-17} \frac{\text{J}}{\text{operation}}$$

Minimal Turing Machine:

A summary



Devices: 314

Area = 75nm × 75nm

Device density: $5.6 \times 10^{12} \text{ cm}^{-2}$

Energy per cycle = $4 \cdot 10^{-18} \frac{J}{\text{cycle}}$

Time per cycle

~2 ps

Power: $2 \mu\text{W}$

Power density : $\sim 30 \text{ kW/cm}^2$

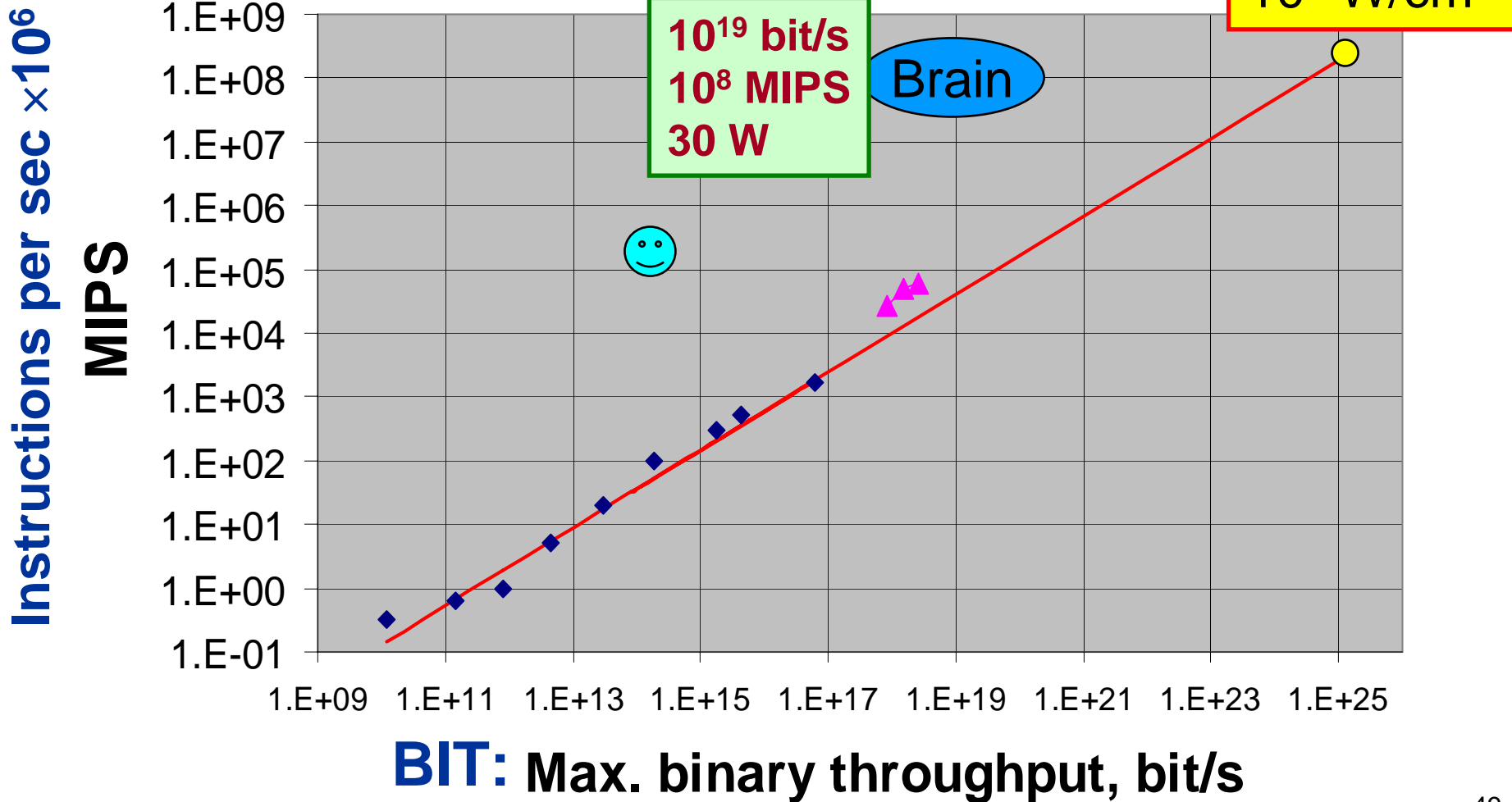
BITS: 10^{14} bit/s

MIPS: 2×10^5

Computing Power: MIPS (μ) vs. BIT (β)



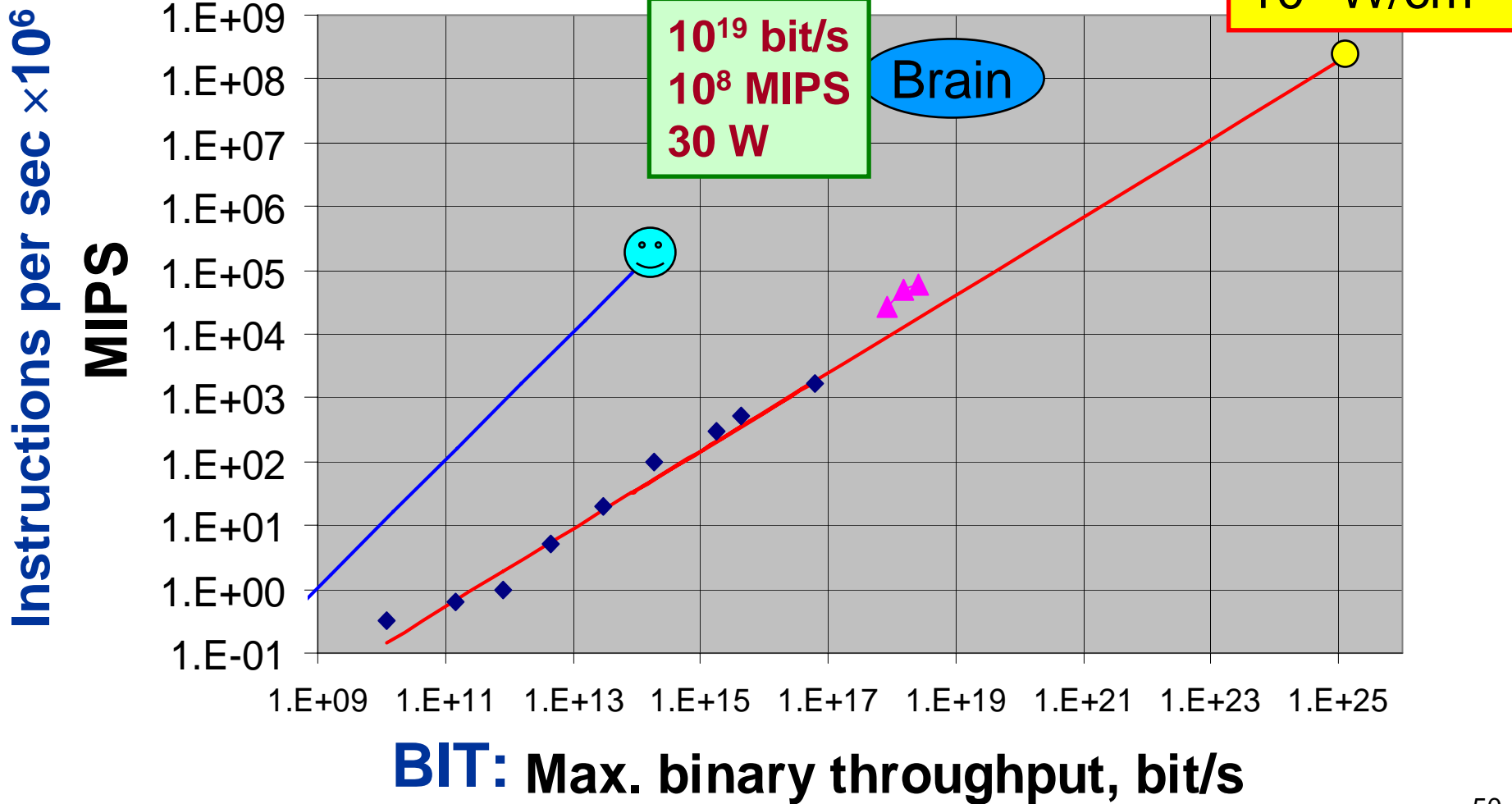
Sources: *The Intel Microprocessor Quick Reference Guide* and *TSCP Benchmark Scores*



Computing Power: MIPS (μ) vs. BIT (β)



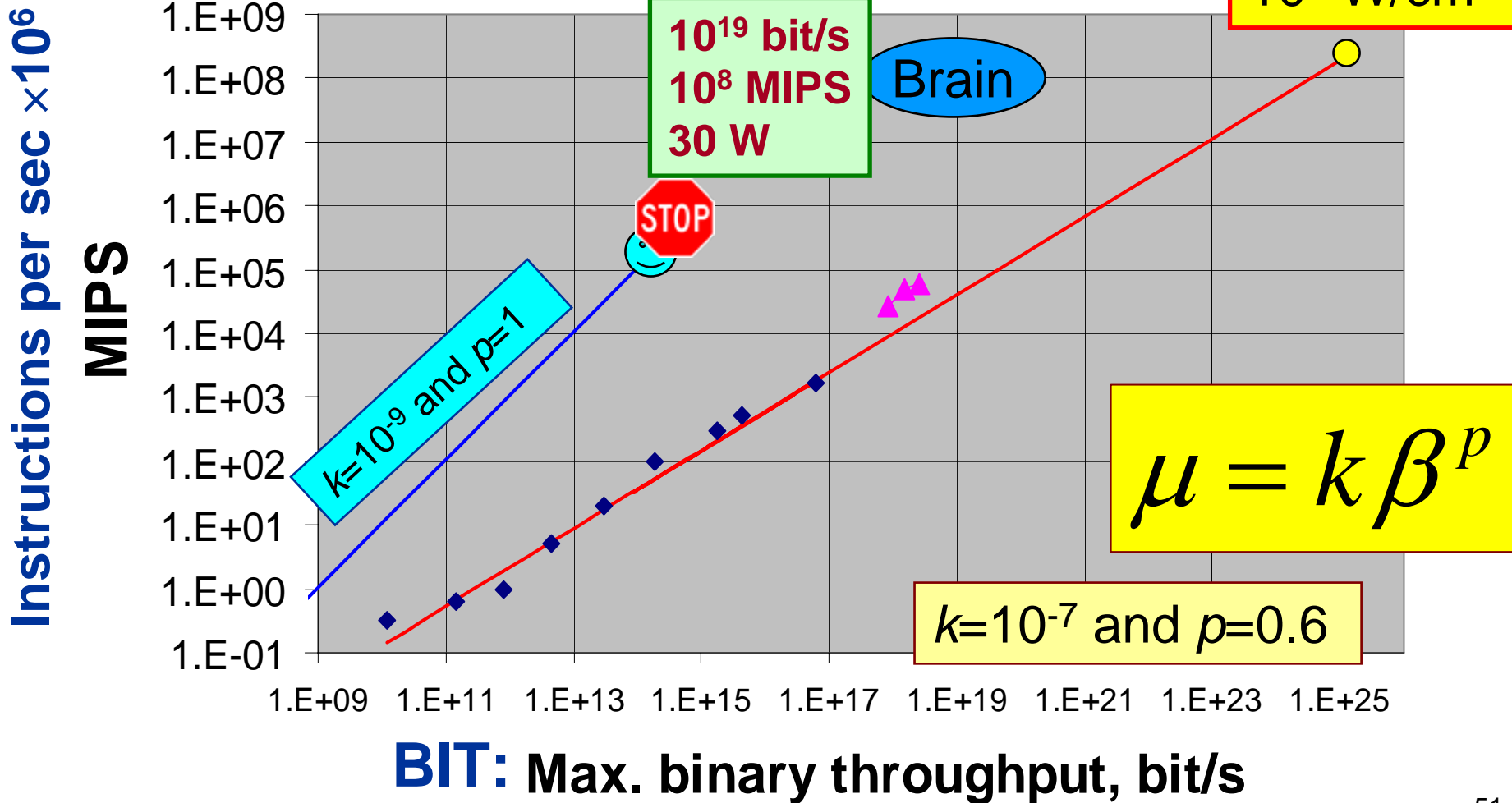
Sources: *The Intel Microprocessor Quick Reference Guide* and *TSCP Benchmark Scores*



Computing Power: MIPS (μ) vs. BIT (β)



Sources: *The Intel Microprocessor Quick Reference Guide* and *TSCP Benchmark Scores*



Summary



- ◆ The Minimal Turing Machine lies on the different performance trajectories from conventional computers
 - ❖ It has slope to meet brain performance

- ◆ More detailed physics based analysis is needed
 - ❖ System thermodynamics of computation
 - **Carnot's equivalent for Computational Engine?**

- ◆ Lessons from Biological Computation?

- ◆ Candidates for beyond-CMOS nano-electronics should be evaluated in the context of system scaling
 - ❖ e.g. spintronic minimal Turing Machine?



Back-up

Extreme Multi-Core Analysis



Power consumption by K cores:

$$P_K = K \cdot \frac{M}{t_{sw}(M)} \cdot E_{b_{\min}}(M) = \frac{N}{t_{sw}(M)} \cdot E_{b_{\min}}(N)$$

