Variability Modeling at the Device Level for Circuit Simulation

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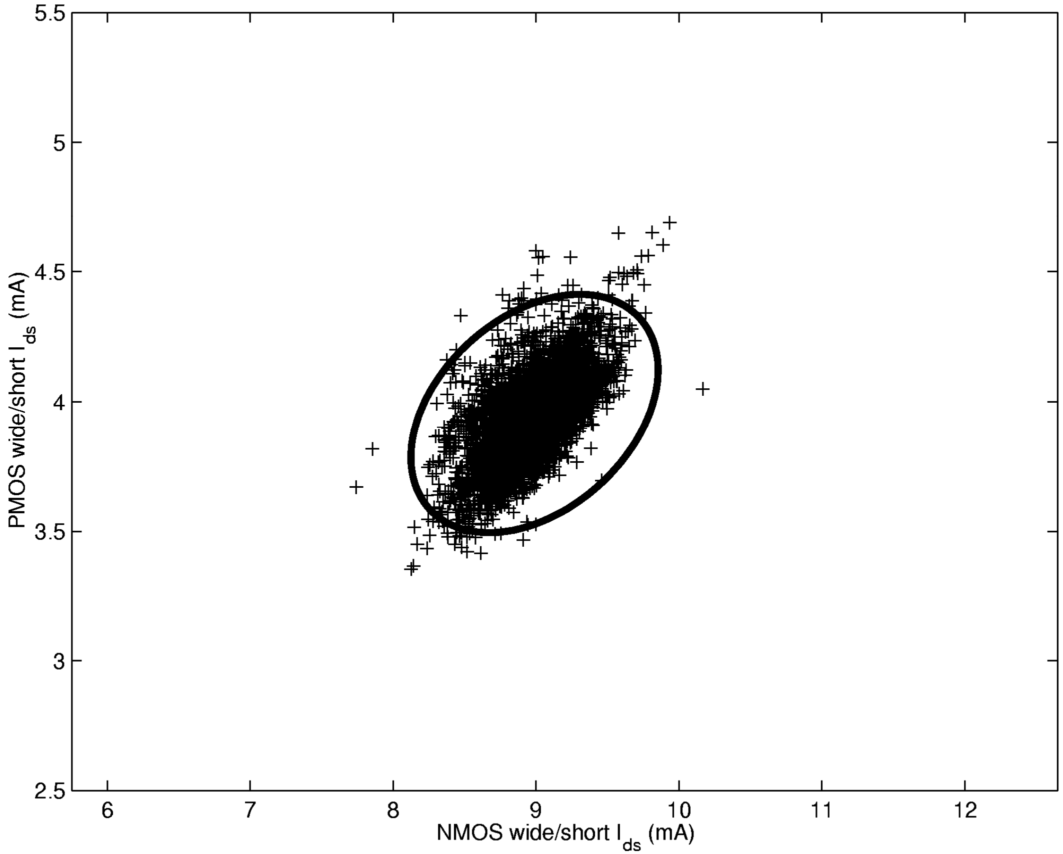
Overview

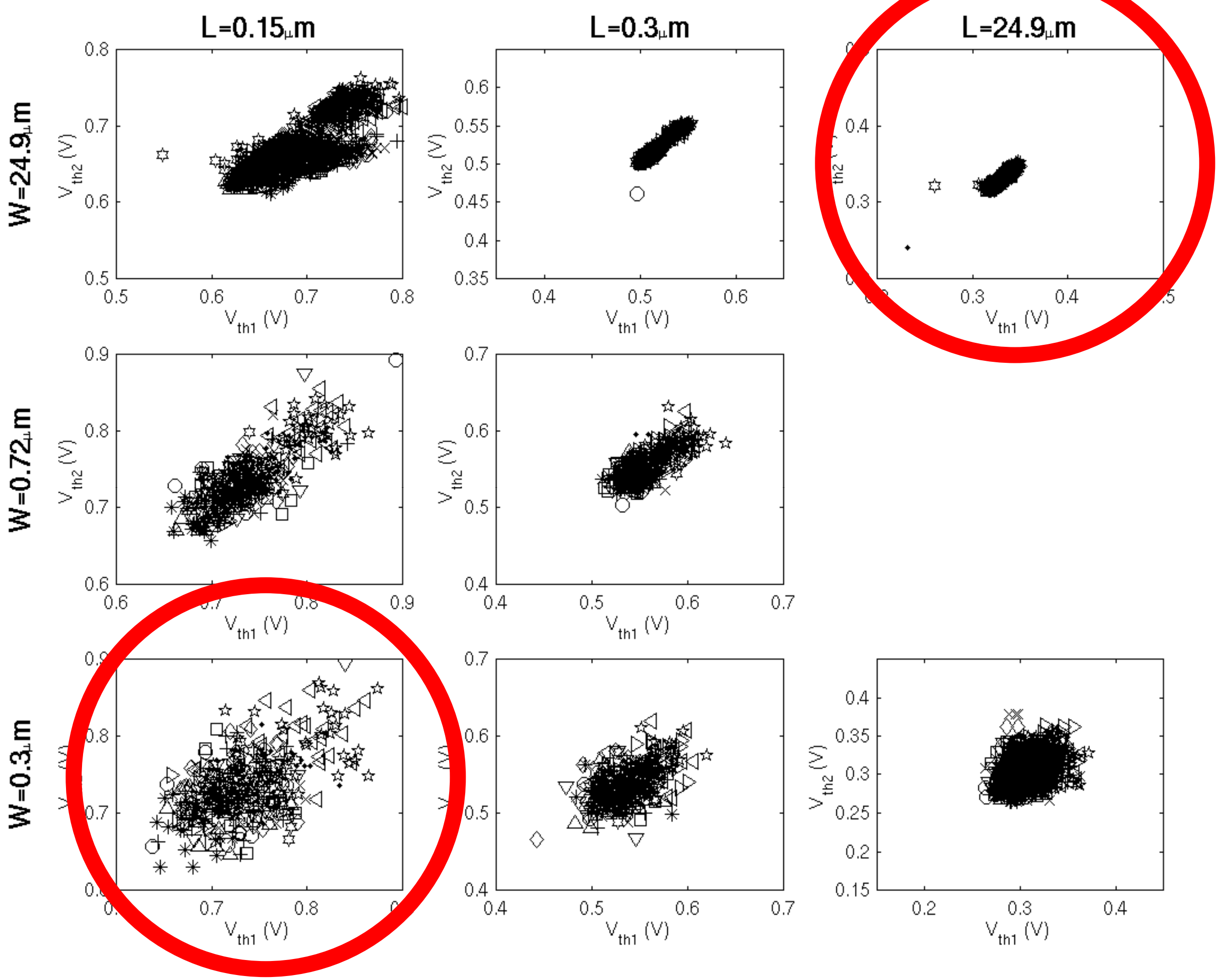
- Introduction
  - statistical simulation
  - modeling basis
  - correlation modeling
- Backward Propagation of Variance (BPV)
  - general procedure
  - consistency for global and local variations
- Examples and application
- Summary

Modeling and Simulating Device Variations

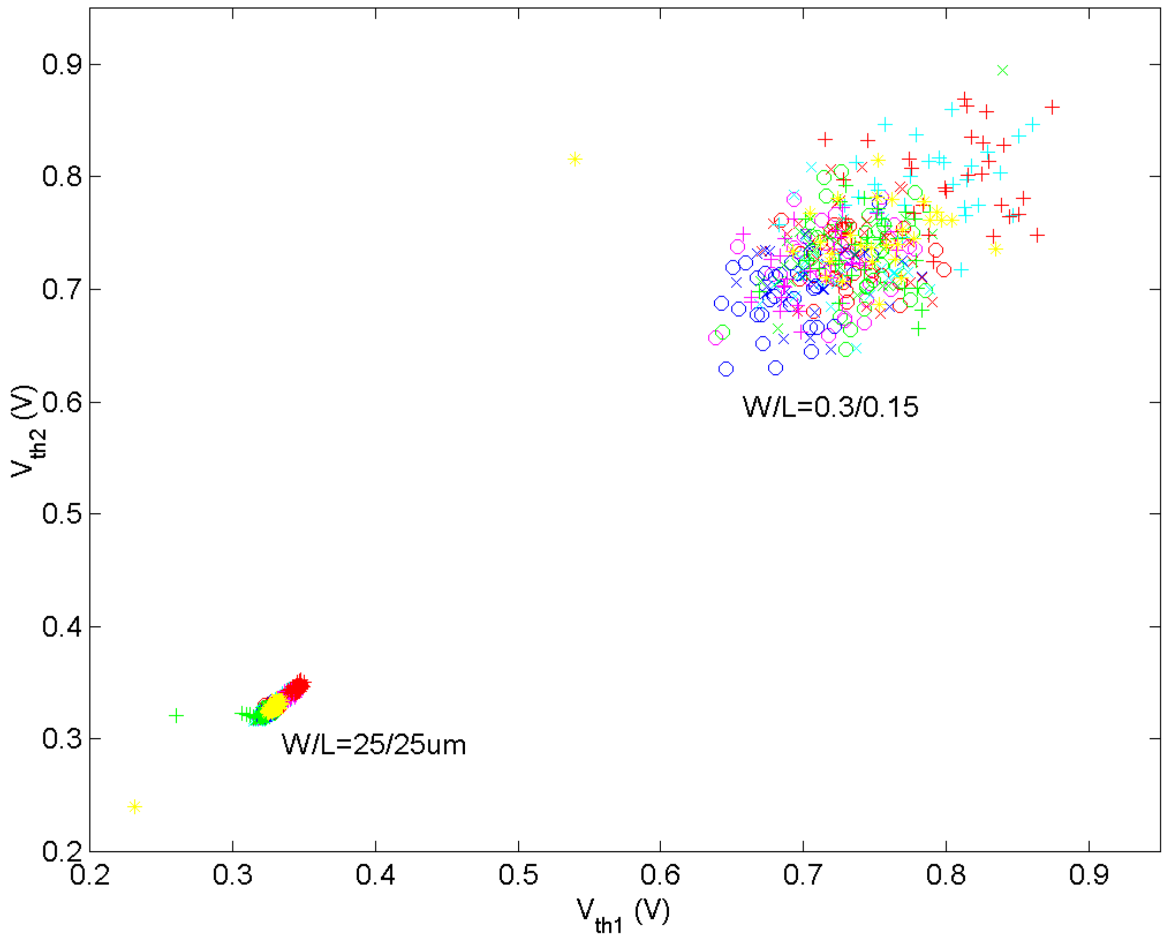


- **Limits (corners)**
- **Distributions**
- **Correlation**
- **Mismatch**

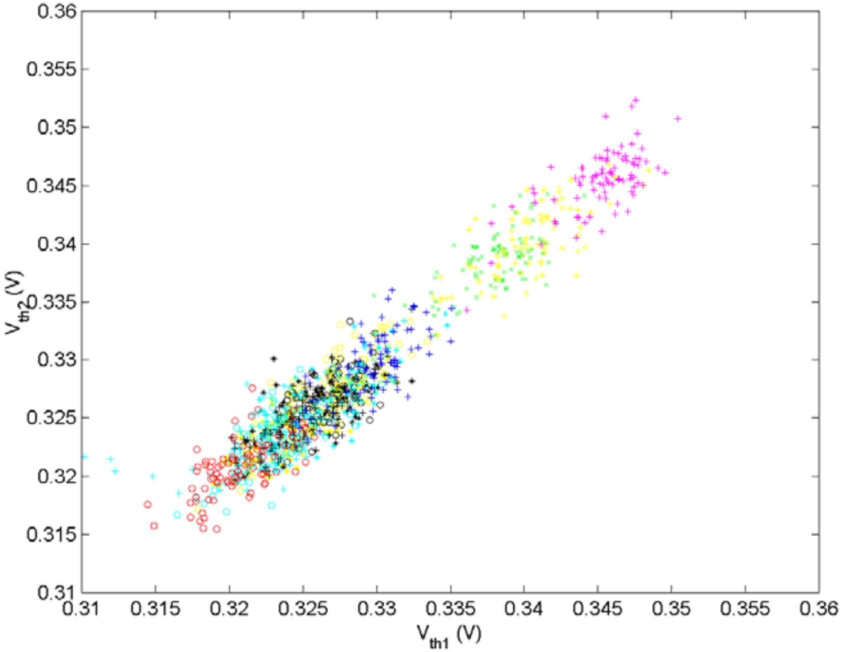
V_{TH} Variation

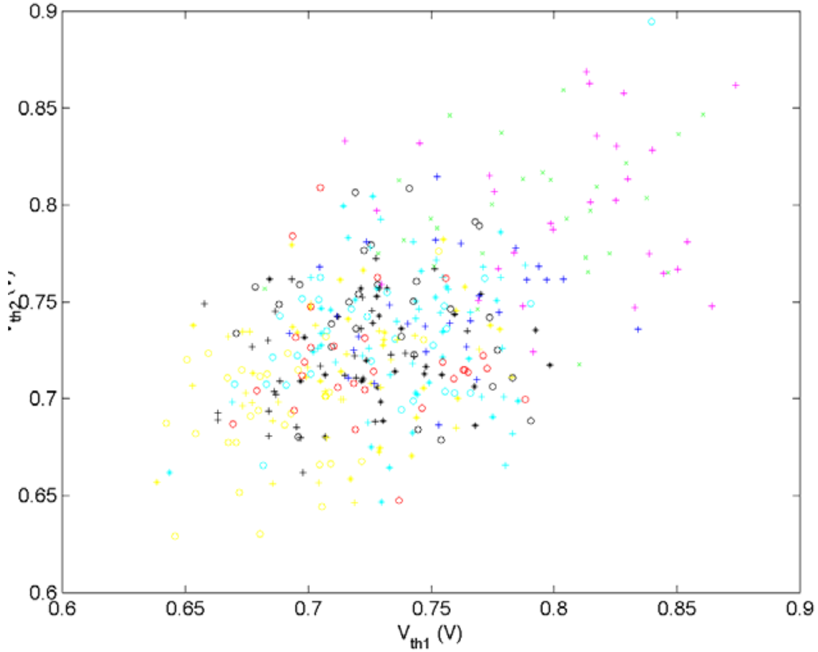


V_{TH} Variation Big and Small devices



V_{TH} Correlation





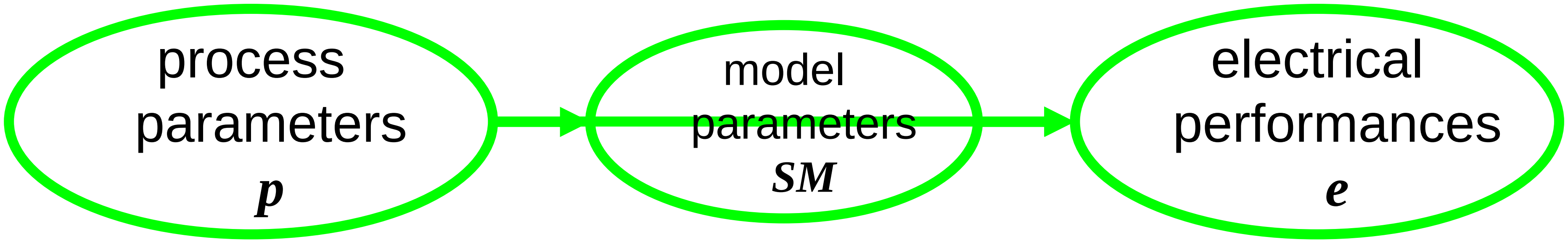
Modeling Variability: Global and Local Variations

$$p_i = \overline{p}_i + n_{sgl}\sigma_{p_i,gl} + n_{smm}\sigma_{p_i,mm}(\vec{g})$$

- Models (partial) correlations using uncorrelated parameters
- Avoid using explicit correlation coefficients between p_i
  - mismatch is **not** “pair-wise” - $O(N^2)$ correlations for N devices!
  - does not capture geometry dependence of p_i variation
  - does not allow mismatch-only simulation
  - does not allow mismatch simulation if global variation is zero
    - > some parameters are included only for mismatch modeling
- Corner simulation models defined by setting n_{sgl}
- Distribution simulation models sample n_{sgl} and/or n_{smm}
  - naturally has good numerical scaling

Statistical Modeling Basis

- Statistical models **must** be based on key physical parameters
  - $\boldsymbol{p}$ fundamentally control device electrical performances $\boldsymbol{e}$
  - components of $\boldsymbol{p}$ are independent
    - > correlations between e_m , e_n from models, parameter mappings
  - normally distributed p_i
    - > log-normal e_m come through exp(p_i) mappings in model files
    - > e.g. BJT base current (or β), MOSFET $R_{S/D}$ series resistance
  - define $\boldsymbol{p} \rightarrow \boldsymbol{SM}$ mappings in model files if needed (BJT)



Statistical Modeling Basis (2)

- Use uncorrelated process parameters $\boldsymbol{p}$
  - V_{fb}, t_{ox}, N_{sub}, μ_0, …
- Do **not** use correlated model parameters $\boldsymbol{SM}$
  - V_{t0}, γ, k', …
- **Always** explains anomalous geometry, bias dependence
- Natural separation into absolute and relative (%) variations
  - scales to ~1 for numerical stability, easy to mentally sanity check
  - V_{fb} and lateral $\Delta_{[LW]}$ absolute, others relative
- Automatically gives correlations
  - between model "parameters" (from $\boldsymbol{p} \rightarrow \boldsymbol{SM}$ mappings)
  - between device electrical performances

Statistical Simulation

- IC design flows can embody different types of statistical design practices

- The types of statistical simulation techniques used dictates the types of statistical models that are required
  - corner simulations require case files
  - MC simulations need distributional models
  - mismatch analysis needs mismatch models
  - sensitivity analysis needs physical models

- Global statistical models do not model σ
  - σ is only known accurately when it is too late to be of use
  - define statistical models in terms of engineering specifications

Mis-Propagation of Variance

- If gain factor β and zero-bias threshold V_{t0} are uncorrelated

$$\delta e_m = \frac{\partial e_m}{\partial V_{t0}} \delta V_{t0} + \frac{\partial e_m}{\partial \beta} \delta \beta$$

$$\sigma_{e_m}^2 = \left(\frac{\partial e_m}{\partial V_{t0}} \right)^2 \sigma_{V_{t0}}^2 + \left(\frac{\partial e_m}{\partial \beta} \right)^2 \sigma_\beta^2$$

$$\beta = \mu_0 C_{ox}^{'} \frac{W}{L} = k^{'} \frac{W}{L}$$

Mis-Propagation of Variance (2)

- In reality V_{t0} and β **are** correlated, e.g. through t_{ox}

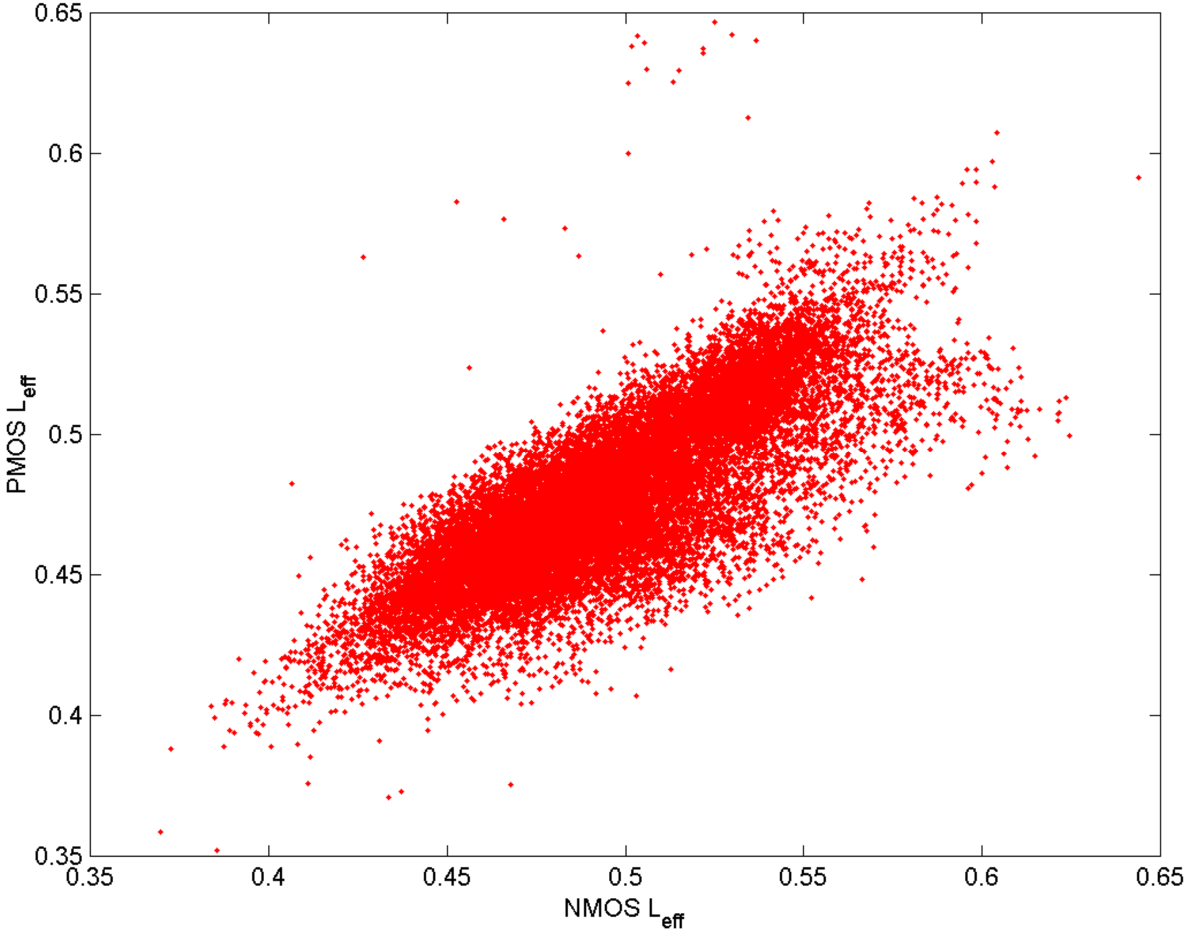
$$\delta e_m = \left(\frac{\partial e_m}{\partial V_{t0}} \frac{\partial V_{t0}}{\partial t_{ox}} + \frac{\partial e_m}{\partial \beta} \frac{\partial \beta}{\partial t_{ox}} \right) \delta t_{ox}$$

$$\sigma_{e_m}^2 = \left(\frac{\partial e_m}{\partial V_{t0}} \frac{\partial V_{t0}}{\partial t_{ox}} + \frac{\partial e_m}{\partial \beta} \frac{\partial \beta}{\partial t_{ox}} \right)^2 \sigma_{t_{ox}}^2$$

$$= \left(\frac{\partial e_m}{\partial V_{t0}} \right)^2 \sigma_{V_{t0}}^2 + \left(\frac{\partial e_m}{\partial \beta} \right)^2 \sigma_{\beta}^2$$

$$+ 2 \frac{\partial e_m}{\partial V_{t0}} \frac{\partial V_{t0}}{\partial t_{ox}} \frac{\partial e_m}{\partial \beta} \frac{\partial \beta}{\partial t_{ox}} \sigma_{t_{ox}}^2$$

Correlation Modeling via Uncorrelated Parameters



L_{eff} Modeling Approach: Analytical Example

- Lump correlated (common) variation into poly CD
  - litho and etch are common between NMOS and PMOS
- Lump uncorrelated (independent) variation into source/drain out-diffusion
  - source/drain implants differ between NMOS and PMOS

$$\Delta_{Lp} = C_d + O_{dp}$$

$$\Delta_{Ln} = C_d + O_{dn}$$

- Big question: how can these separate components be characterized? Start by forming

$$\Delta_{\Delta L} = \Delta_{Lp} - \Delta_{Ln}$$

Statistical L_{eff} Modeling

- At first it seems there is no new information in $\Delta_{\Delta L}$ over that in the individual NMOS and PMOS Δ_L values
- But statistically there is

$$\begin{bmatrix} \Delta_{Lp} \\ \Delta_{Ln} \\ \Delta_{\Delta L} \end{bmatrix} = \begin{bmatrix} +1 & 0 & +1 \\ 0 & +1 & +1 \\ +1 & -1 & 0 \end{bmatrix} \begin{bmatrix} O_{dp} \\ O_{dn} \\ C_d \end{bmatrix}$$

singular

$$\begin{bmatrix} \sigma^2_{\Delta Lp} \\ \sigma^2_{\Delta Ln} \\ \sigma^2_{\Delta\Delta L} \end{bmatrix} = \begin{bmatrix} +1 & 0 & +1 \\ 0 & +1 & +1 \\ +1 & +1 & 0 \end{bmatrix} \begin{bmatrix} \sigma^2_{Odp} \\ \sigma^2_{Odn} \\ \sigma^2_{Cd} \end{bmatrix}$$

non-singular

Statistical L_{eff} Characterization

- From measured data calculate the variances of Δ_{Lp}, Δ_{Ln}, and $\Delta_{\Delta L}$, then

$$\begin{bmatrix} \sigma_{Odp}^2 \\ \sigma_{Odn}^2 \\ \sigma_{Cd}^2 \end{bmatrix} = 0.5 \begin{bmatrix} 1 & -1 & 1 \\ -1 & 1 & 1 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} \sigma_{\Delta Lp}^2 \\ \sigma_{\Delta Ln}^2 \\ \sigma_{\Delta\Delta L}^2 \end{bmatrix}$$

- There are only 2 measurements, how come we get 3 pieces of information from those?

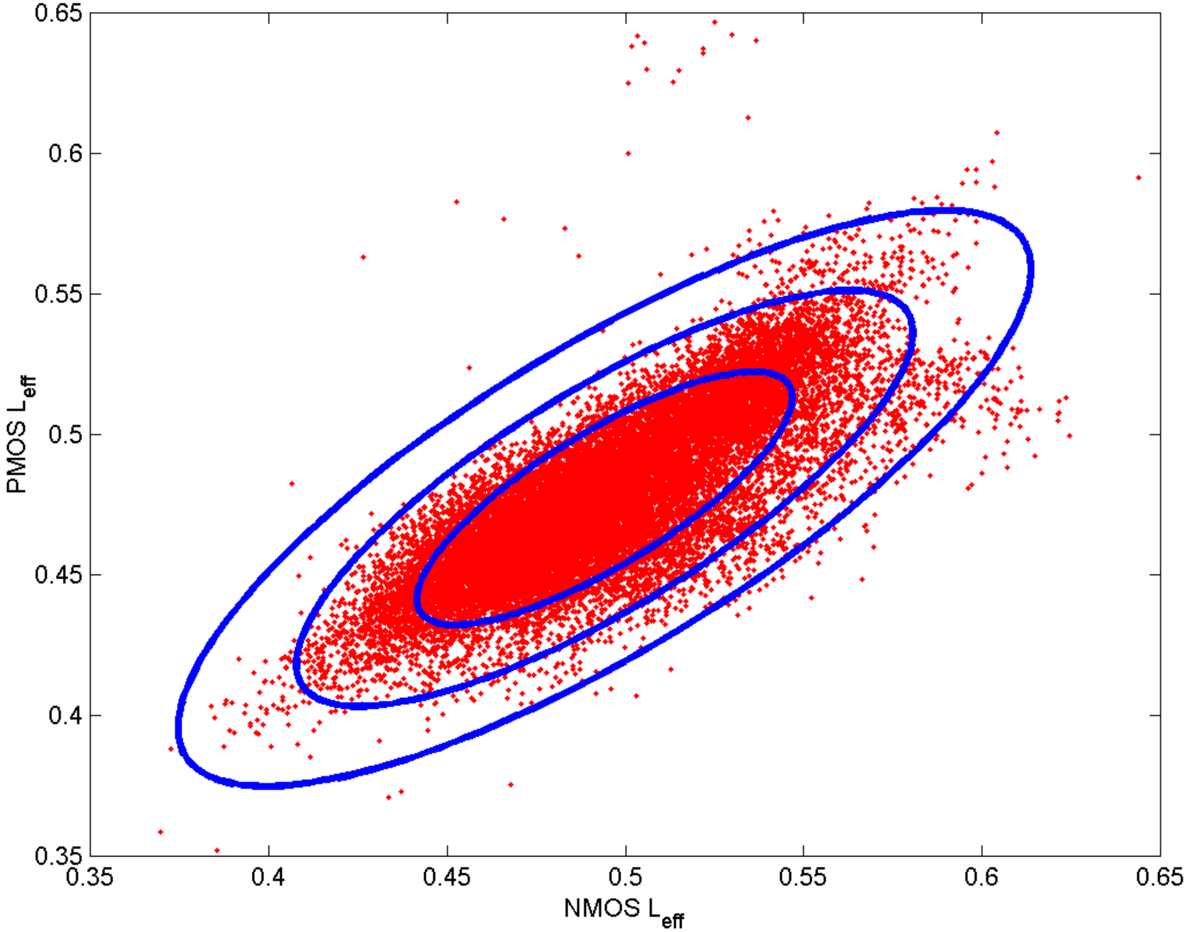
$$\rho_{LpLn} = \frac{\sigma_{Cd}^2}{\sqrt{(\sigma_{Odp}^2 + \sigma_{Cd}^2)(\sigma_{Odn}^2 + \sigma_{Cd}^2)}}$$

Statistical L_{eff} Monte Carlo Simulation

- 10,000 Monte Carlo (MC) samples of O_{dp}, O_{dn}, C_d

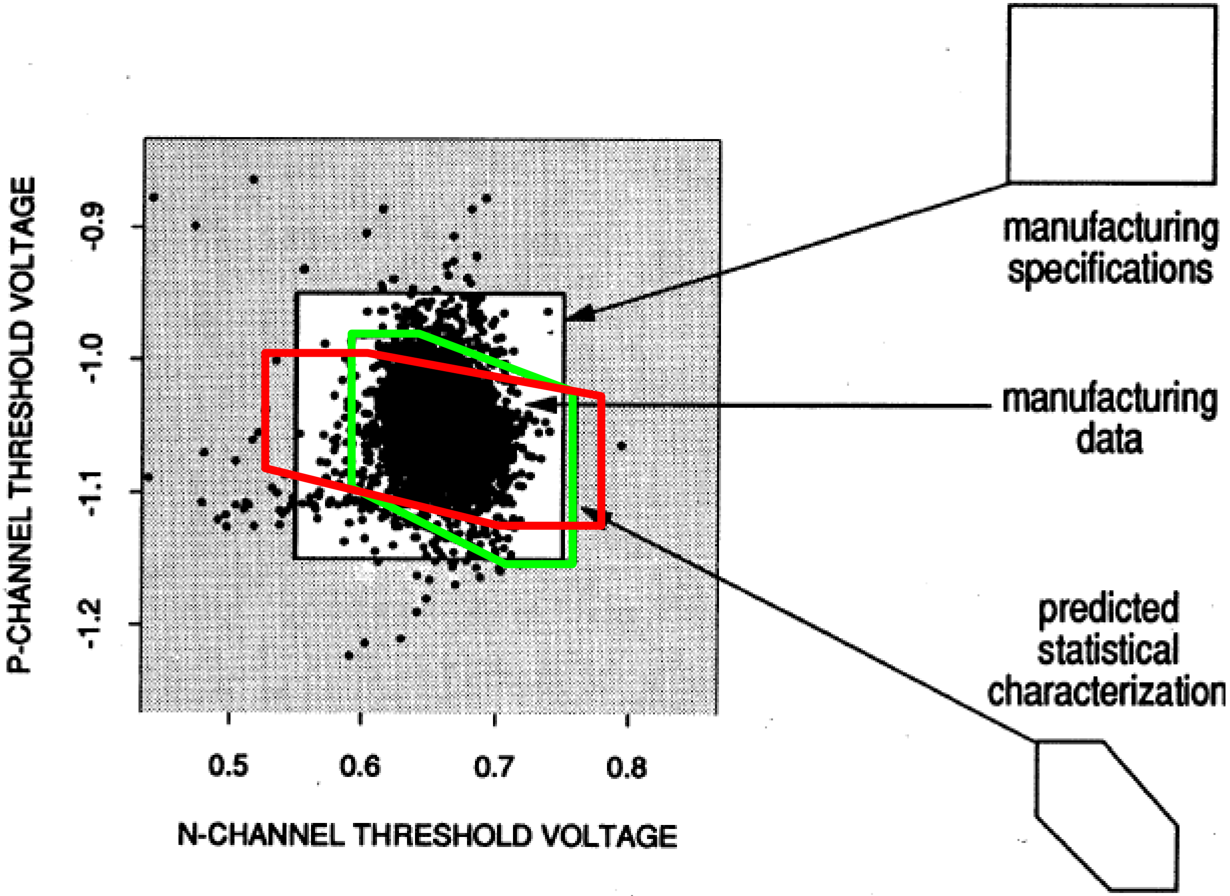
Measured	MC Simulation	Modeled
$\sigma_{\Delta Lp}$=0.02999	$\sigma_{\Delta Lp}$=0.02973	σ_{Odp}=0.00771
$\sigma_{\Delta Ln}$=0.03518	$\sigma_{\Delta Ln}$=0.03473	σ_{Odn}=0.01993
$\sigma_{\Delta\Delta L}$=0.02137	$\sigma_{\Delta\Delta L}$=0.02106	σ_{Cd}=0.02899
ρ_{LpLn}=0.7964	ρ_{LpLn}=0.7972	ρ_{LpLn}=0.7964

Statistical L_{eff} MC Compared to Measured Data



What is the Goal of Statistical Device Modeling?

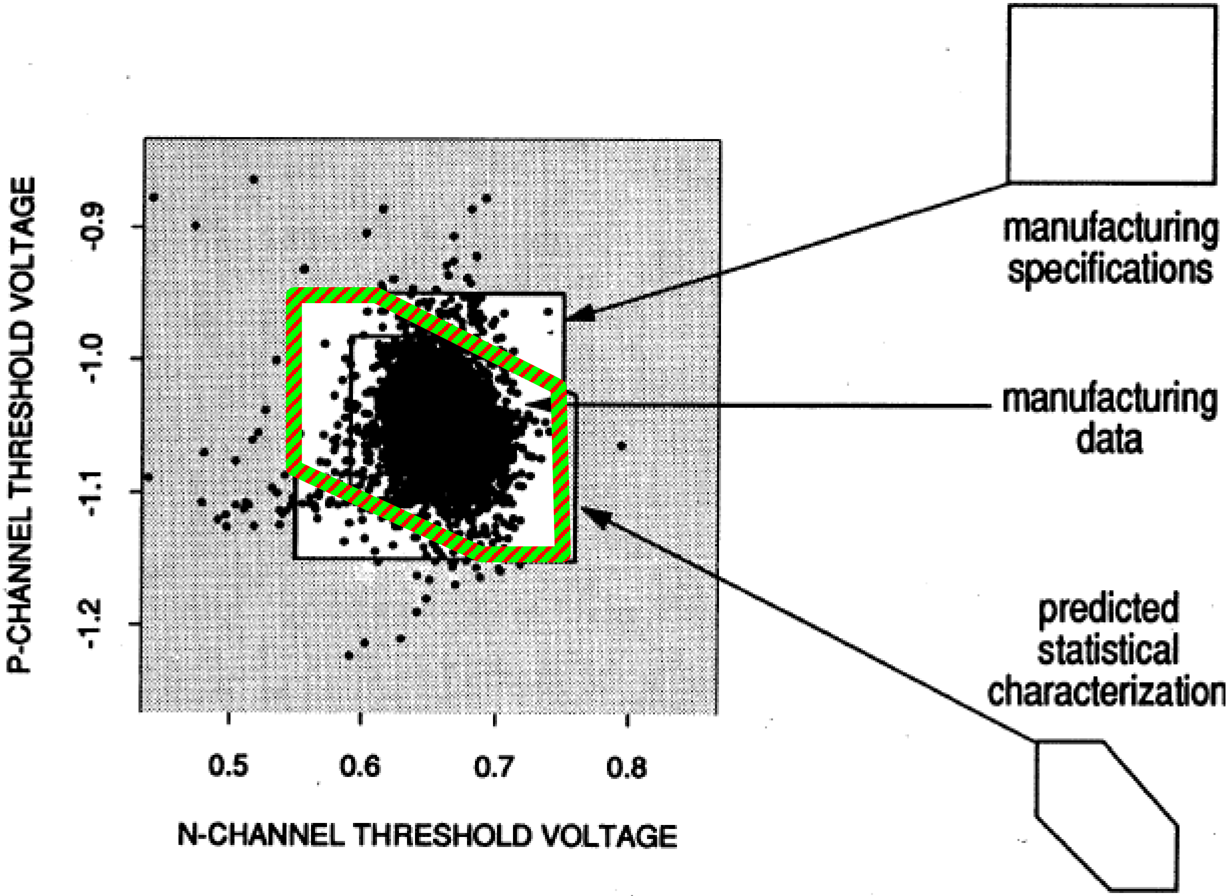
Model A
Model D



The Goal of Statistical Device Modeling!

Model A

Model D



Extreme Case Characterization

- Generate data sets from TCAD simulations or “corner” lots
- Extract separate model parameter files from the data sets

- Only gives corner files, not distributional models
- Perturbations used to generate corner data
  - not complete
  - not accurate
- Not easily updated after process changes
- Not predictive, must be redone for a new process

Numerical Approaches

- Principle Component Analysis (PCA) best known
  - based on statistical extraction of model parameters
- Easy for EDA companies to provide generic software
- Can be very expensive to generate
  - original methods required complete SPICE model extraction
  - later approaches based on PCM data more efficient
- Sensitive to statistical "noise" in parameter extraction
- No physical basis or insight
- Not easily updated after process changes
- Not predictive, must be redone for a new process

Forward Propagation of Variation (FPV)

- Directly measure variations in $\boldsymbol{p}$

- Gives both distributional and corner models

- Cannot always measure p_i directly
- Test structure or biasing can be very different from typical circuit use
- Different methods to measure p_i can give different values
- Using the same p_i variation in different models gives different e_m variation!
- Variation in e_m depends on number of p_i as well as amount of perturbation
- No accounting for sensitivity $\partial e_m / \partial p_i$
  - variation in e_m totally uncontrolled

Backward Propagation of Variation (BPV)

- Goal is to model **electrical performances**, not parameters
- Based on sensitivity analysis

$$e_m(\boldsymbol{p}) = e_m(\overline{\boldsymbol{p}}) + \sum_i s_{m,i}\delta p_i + \sum_{i,j} s_{m,ij}\delta p_i \delta p_j$$

$$s_{m,i} = \left(\frac{\partial e_m}{\partial p_i}\right)_{\boldsymbol{p}=\overline{\boldsymbol{p}}} \qquad s_{m,ij} = \frac{1}{2}\left(\frac{\partial^2 e_m}{\partial p_i \partial p_j}\right)_{\boldsymbol{p}=\overline{\boldsymbol{p}}} \qquad \delta p_i = p_i - \overline{p}_i$$

BPV in a Nutshell

$$\mu_{e_m} = e_m(\overline{\boldsymbol{p}}) + \sum_i s_{m,ii}\sigma_i^2$$

$$\sigma_{e_m}^2 = \sum_i \left(s_{m,i}^2 + 2\sum_j s_{m,ij}^2 \sigma_j^2 \right)\sigma_i^2$$

$$\gamma_{e_m} = \frac{1}{\sigma_{e_m}^3} \sum_{i,j} \left(6 s_{m,i} s_{m,j} s_{m,ij} + 8\sum_k s_{m,ij} s_{m,jk} s_{m,ki} \sigma_k^2 \right)\sigma_i^2 \sigma_j^2$$

$$\sigma_{e_m,e_n} = \sum_i \left(s_{m,i} s_{n,i} + 2\sum_j s_{m,ij} s_{n,ij} \sigma_j^2 \right)\sigma_i^2$$

Linear BPV

$$\mu_{e_m} = e_m(\bar{\boldsymbol{p}})$$

$$\sigma_{e_m}^2 = \sum_i s_{m,i}^2 \sigma_i^2$$

$$\sigma_{e_m,e_n} = \sum_i s_{m,i} s_{n,i} \sigma_i^2$$

BPV Procedure

- Sensitivities are computed from SPICE models
  - different values for different models (e.g. PSP and BSIM4)
- Mean, variance, skewness, and correlation of e based on manufacturing data
  - adjusted using engineering knowledge to define specs
  - skewness and correlation not needed for all performances
  - correlation often important
  - nonlinearity rarely important (occasionally for BJTs)
- BPV equations are solved for the mean and variance of the process parameters
  - one or more FPV parameters can also be included
  - if nonlinearities are small mean p can be directly computed using nonlinear least-squares optimization as a separate step

Qualitative Evaluation

- Perception
  - looks hopelessly complex
  - “math” scares engineers
- Reality
  - when you get past the “math” and see what is really happening it is ***incredibly*** simple
  - almost cheating
    - > “forces” statistics in p_i to fit observed variations in e_m
  - remember: the goal of modeling is to accurately represent e_m

Mathematical Details

- The whole process falls in a heap if
  - underlying basic SPICE models are inaccurate
    - > sensitivities get messed up
  - there are inconsistencies in specification of the statistics
    - > variances can become negative
  - measurements are not selected wisely
    - > the matrix of squared sensitivities becomes ill conditioned
- These are very **good** things
  - not a “garbage in garbage out” process
    - > has to be well posed to give results
  - detects problems in the above 3 areas
- Have not specified what “type” of variances
  - same approach works for global and local (mismatch) variation
  - performances can be device or circuit level

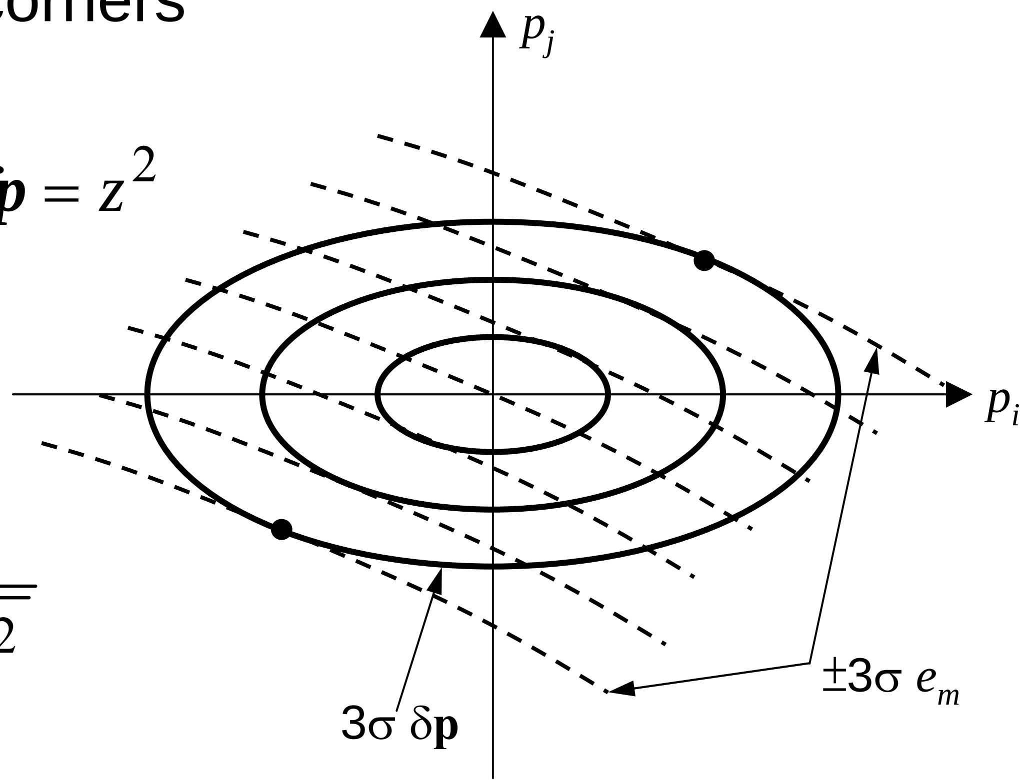
Corner Models from BPV

- Specify generic corners in terms of e_m, not p_i
  - **must** use physical knowledge to avoid inconsistency
  - easily solve using nonlinear least-squares optimizer
- Can also compute exact corners for a specific e_m

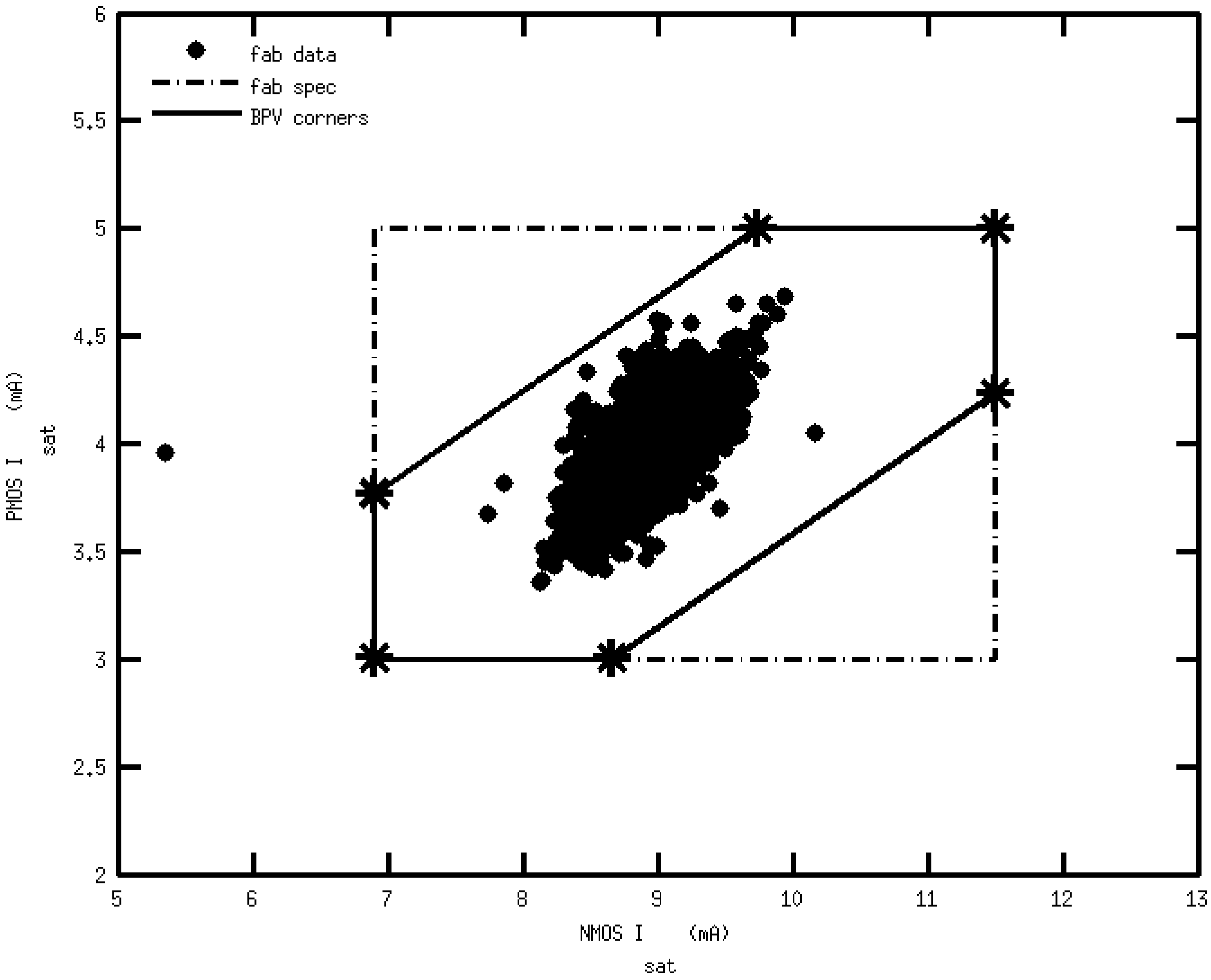
$$\max_{\delta \mathbf{p}} \delta e_m \text{ subject to } \delta \mathbf{p}^T C^{-1} \delta \mathbf{p} = z^2$$

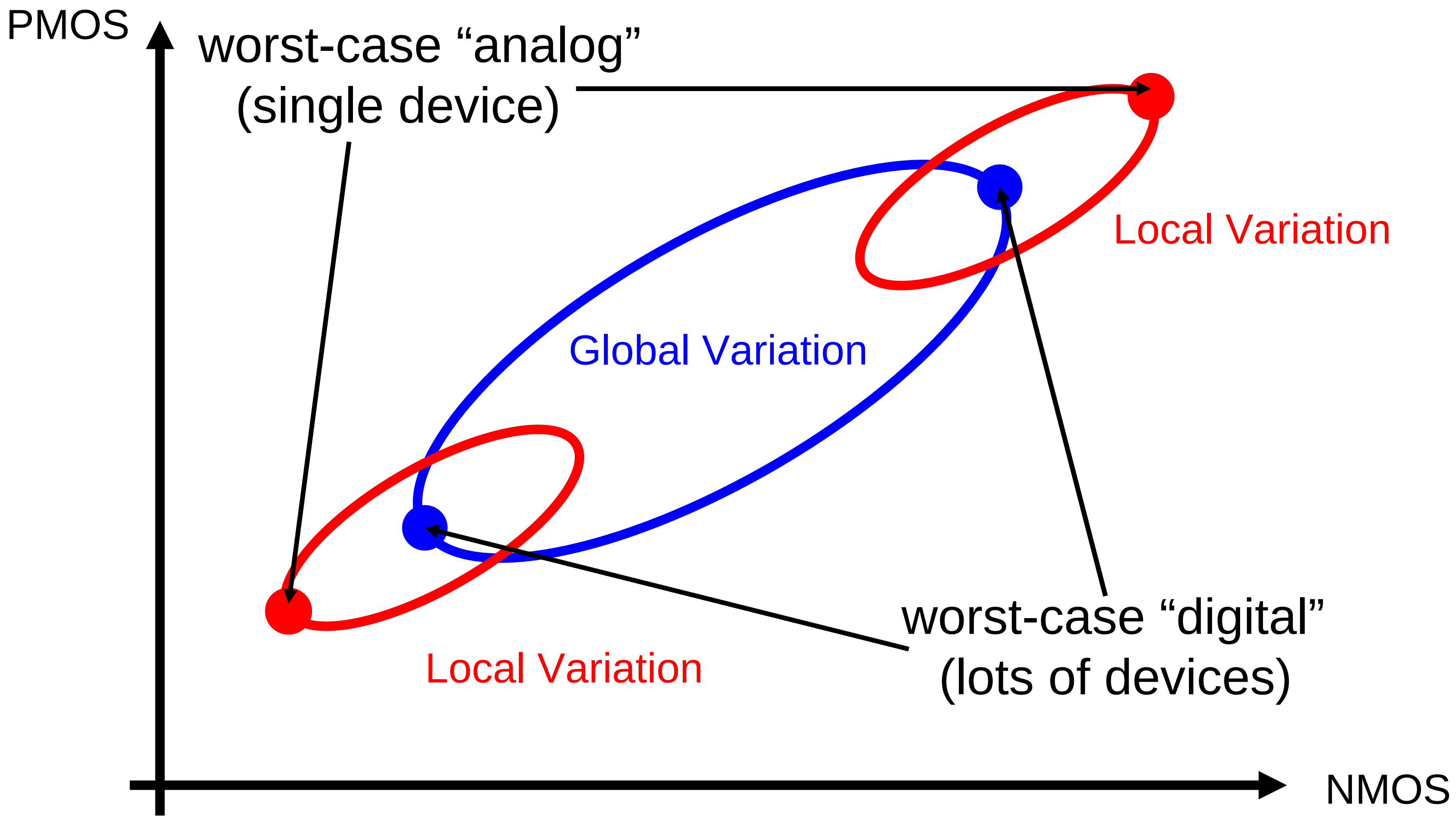
$$\delta \mathbf{p} = \pm z \frac{Cs}{\sqrt{s^T Cs}}, s = \frac{\partial e_m}{\partial \mathbf{p}}$$

$$\delta p_i = \pm z \frac{\sigma_i^2 (\partial e_m / p_i)}{\sqrt{\sum_j \sigma_j^2 (\partial e_m / p_j)^2}}$$



BPV Corner Models




PMOS
worst-case "analog"
(single device)
Local Variation
Global Variation
Local Variation
worst-case "digital"
(lots of devices)
NMOS

MOSFET Example: BSIM3 Distributional Modeling

$$\begin{bmatrix} \sigma^2_{\delta V_{tr}} \\ \sigma^2_{\frac{\delta\beta_r}{\beta_r}} \\ \sigma^2_{\delta V_{ts}} \\ \sigma^2_{\frac{\delta I_{ss}}{I_{ss}}} \end{bmatrix} = \begin{bmatrix} \left(t_{ox}\frac{\partial V_{tr}}{\partial t_{ox}}\right)^2 & \left(\frac{\partial V_{tr}}{\partial V_{fb}}\right)^2 & \left(\mu_0\frac{\partial V_{tr}}{\partial \mu_0}\right)^2 & \left(\frac{\partial V_{tr}}{\partial \Delta_L}\right)^2 & \left(\frac{\partial V_{tr}}{\partial V_{tl}}\right)^2 \\ \left(\frac{t_{ox}}{\beta_r}\frac{\partial \beta_r}{\partial t_{ox}}\right)^2 & \left(\frac{1}{\beta_r}\frac{\partial \beta_r}{\partial V_{fb}}\right)^2 & \left(\frac{\mu_0}{\beta_r}\frac{\partial \beta_r}{\partial \mu_0}\right)^2 & \left(\frac{1}{\beta_r}\frac{\partial \beta_r}{\partial \Delta_L}\right)^2 & \left(\frac{1}{\beta_r}\frac{\partial \beta_r}{\partial V_{tl}}\right)^2 \\ \left(t_{ox}\frac{\partial V_{ts}}{\partial t_{ox}}\right)^2 & \left(\frac{\partial V_{ts}}{\partial V_{fb}}\right)^2 & \left(\mu_0\frac{\partial V_{ts}}{\partial \mu_0}\right)^2 & \left(\frac{\partial V_{ts}}{\partial \Delta_L}\right)^2 & \left(\frac{\partial V_{ts}}{\partial V_{tl}}\right)^2 \\ \left(\frac{t_{ox}}{I_{ss}}\frac{\partial I_{ss}}{\partial t_{ox}}\right)^2 & \left(\frac{1}{I_{ss}}\frac{\partial I_{ss}}{\partial V_{fb}}\right)^2 & \left(\frac{\mu_0}{I_{ss}}\frac{\partial I_{ss}}{\partial \mu_0}\right)^2 & \left(\frac{1}{I_{ss}}\frac{\partial I_{ss}}{\partial \Delta_L}\right)^2 & \left(\frac{1}{I_{ss}}\frac{\partial I_{ss}}{\partial V_{tl}}\right)^2 \end{bmatrix} \begin{bmatrix} \sigma^2_{\frac{\delta t_{ox}}{t_{ox}}} \\ \sigma^2_{\delta V_{fb}} \\ \sigma^2_{\frac{\delta\mu_0}{\mu_0}} \\ \sigma^2_{\delta\Delta_L} \\ \sigma^2_{\delta V_{tl}} \end{bmatrix}$$

$$\sigma^2_{e_m} = \sum_i \left(\frac{\partial e_m}{\partial p_i}\right)^2 \sigma^2_{p_i}$$

Mixed FPV and BPV

$$
\begin{bmatrix}
\sigma^2_{\delta V_{tr}} - \left(t_{ox}\frac{\partial V_{tr}}{\partial t_{ox}}\right)^2 \sigma^2_{\frac{\delta t_{ox}}{t_{ox}}} \\
\sigma^2_{\frac{\delta \beta_r}{\beta_r}} - \left(\frac{t_{ox}}{\beta_r}\frac{\partial \beta_r}{\partial t_{ox}}\right)^2 \sigma^2_{\frac{\delta t_{ox}}{t_{ox}}} \\
\sigma^2_{\delta V_{ts}} - \left(t_{ox}\frac{\partial V_{ts}}{\partial t_{ox}}\right)^2 \sigma^2_{\frac{\delta t_{ox}}{t_{ox}}} \\
\sigma^2_{\frac{\delta I_{ss}}{I_{ss}}} - \left(\frac{t_{ox}}{I_{ss}}\frac{\partial I_{ss}}{\partial t_{ox}}\right)^2 \sigma^2_{\frac{\delta t_{ox}}{t_{ox}}}
\end{bmatrix}
=
\begin{bmatrix}
\left(\frac{\partial V_{tr}}{\partial V_{fb}}\right)^2 & \left(\mu_0\frac{\partial V_{tr}}{\partial \mu_0}\right)^2 & \left(\frac{\partial V_{tr}}{\partial \Delta_L}\right)^2 & \left(\frac{\partial V_{tr}}{\partial V_{tl}}\right)^2 \\
\left(\frac{1}{\beta_r}\frac{\partial \beta_r}{\partial V_{fb}}\right)^2 & \left(\frac{\mu_0}{\beta_r}\frac{\partial \beta_r}{\partial \mu_0}\right)^2 & \left(\frac{1}{\beta_r}\frac{\partial \beta_r}{\partial \Delta_L}\right)^2 & \left(\frac{1}{\beta_r}\frac{\partial \beta_r}{\partial V_{tl}}\right)^2 \\
\left(\frac{\partial V_{ts}}{\partial V_{fb}}\right)^2 & \left(\mu_0\frac{\partial V_{ts}}{\partial \mu_0}\right)^2 & \left(\frac{\partial V_{ts}}{\partial \Delta_L}\right)^2 & \left(\frac{\partial V_{ts}}{\partial V_{tl}}\right)^2 \\
\left(\frac{1}{I_{ss}}\frac{\partial I_{ss}}{\partial V_{fb}}\right)^2 & \left(\frac{\mu_0}{I_{ss}}\frac{\partial I_{ss}}{\partial \mu_0}\right)^2 & \left(\frac{1}{I_{ss}}\frac{\partial I_{ss}}{\partial \Delta_L}\right)^2 & \left(\frac{1}{I_{ss}}\frac{\partial I_{ss}}{\partial V_{tl}}\right)^2
\end{bmatrix}
\begin{bmatrix}
\sigma^2_{\delta V_{fb}} \\
\sigma^2_{\frac{\delta \mu_0}{\mu_0}} \\
\sigma^2_{\delta \Delta_L} \\
\sigma^2_{\delta V_{tl}}
\end{bmatrix}
$$

$$
\begin{bmatrix}
\mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\
\mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} \\
\mathbf{1} & \mathbf{0} & \mathbf{a} & \mathbf{b} \\
\mathbf{0} & \mathbf{\sim 1} & \mathbf{c} & \mathbf{s}
\end{bmatrix}
$$

Correlation for BSIM3: 10,000 Sample MC

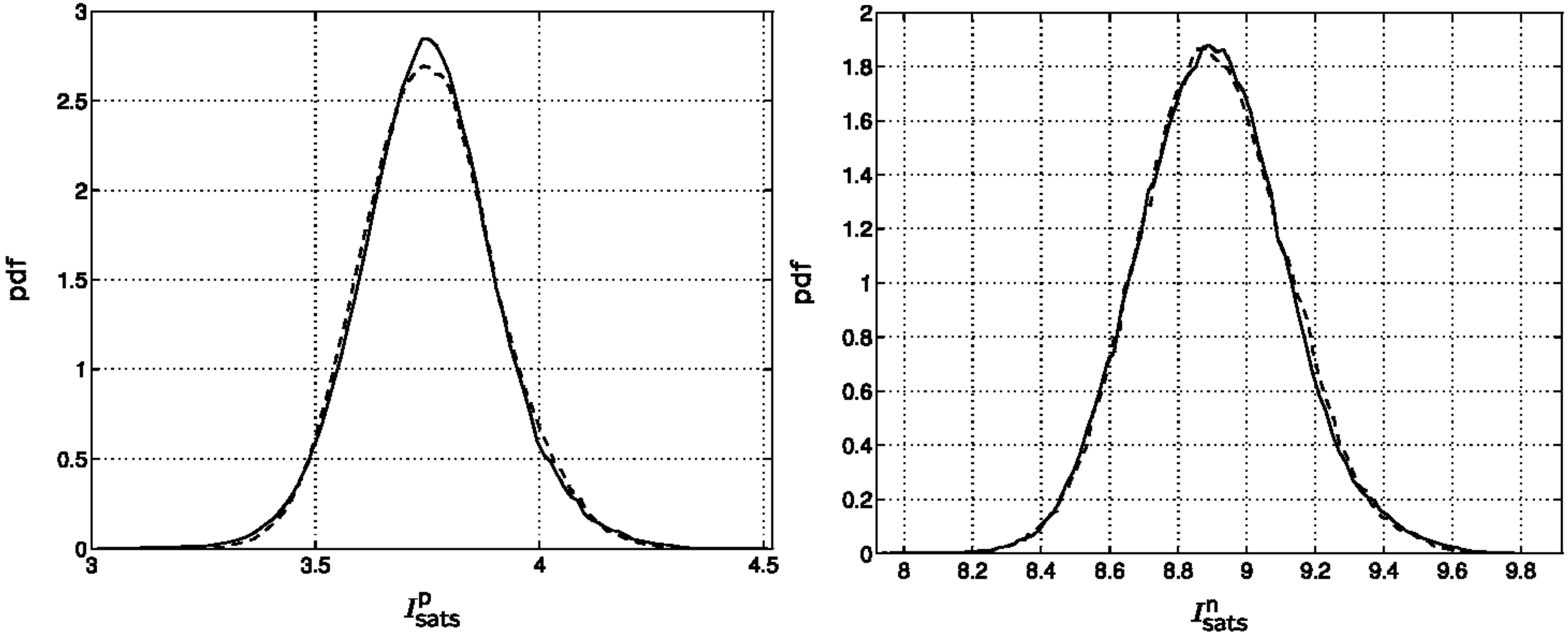
	Targets	
	μ	σ
$V_{\text{th0r}}^{\text{p}}$	0.531	0.0149
$V_{\text{th0s}}^{\text{p}}$	0.509	0.0166
$\beta_{\text{0r}}^{\text{p}}$	34.4	1.8%
$I_{\text{sats}}^{\text{p}}$	3.75	3.9%
$V_{\text{th0r}}^{\text{n}}$	0.496	0.0091
$V_{\text{th0s}}^{\text{n}}$	0.536	0.0100
$\beta_{\text{0r}}^{\text{n}}$	143.9	1.7%
$I_{\text{sats}}^{\text{n}}$	8.9	2.4%
$\rho(\beta_{\text{0r}}^{\text{p}}, \beta_{\text{0r}}^{\text{n}})$	0.596	
$\rho(I_{\text{sats}}^{\text{p}}, I_{\text{sats}}^{\text{n}})$	0.718	

	BPV Model	
	μ	σ
V_{fb}^{p}	-0.12	0.0148
V_{tl}^{p}	2.44	0.686
U_{bref}^{p}	1.16	1.1%
O_{d}^{p}	0.14	0.011
V_{fb}^{n}	-0.035	0.0088
V_{tl}^{n}	0.168	0.0007
U_{bref}^{n}	0.984	1.04%
O_{d}^{n}	0.007	0.008
C_{d}	0.04	0.017
t_{ox}	1.0^{*}	1.43%

	MC from BPV Model (10 000)	
	μ	σ
$V_{\text{th0r}}^{\text{p}}$	0.531	0.0147
$V_{\text{th0s}}^{\text{p}}$	0.509	0.0165
$\beta_{\text{0r}}^{\text{p}}$	34.4	1.8%
$I_{\text{sats}}^{\text{p}}$	3.76	4.1%
$V_{\text{th0r}}^{\text{n}}$	0.496	0.0088
$V_{\text{th0s}}^{\text{n}}$	0.536	0.0102
$\beta_{\text{0r}}^{\text{n}}$	143.9	1.7%
$I_{\text{sats}}^{\text{n}}$	8.9	2.4%
$\rho(\beta_{\text{0r}}^{\text{p}}, \beta_{\text{0r}}^{\text{n}})$	0.594	
$\rho(I_{\text{sats}}^{\text{p}}, I_{\text{sats}}^{\text{n}})$	0.719	

t_{ox} is FPV

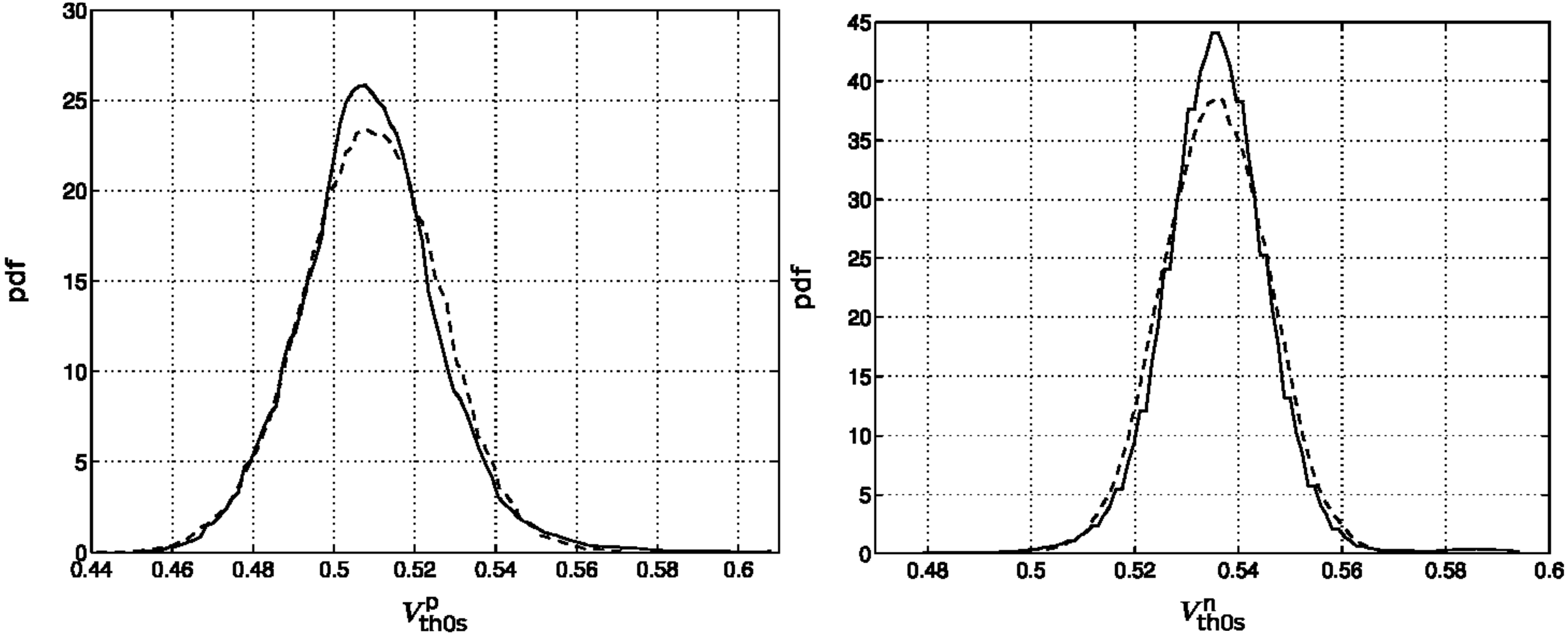
Wide/Short Saturated Drain Current PDFs



dashed: MC from BPV Model

solid: Measurements

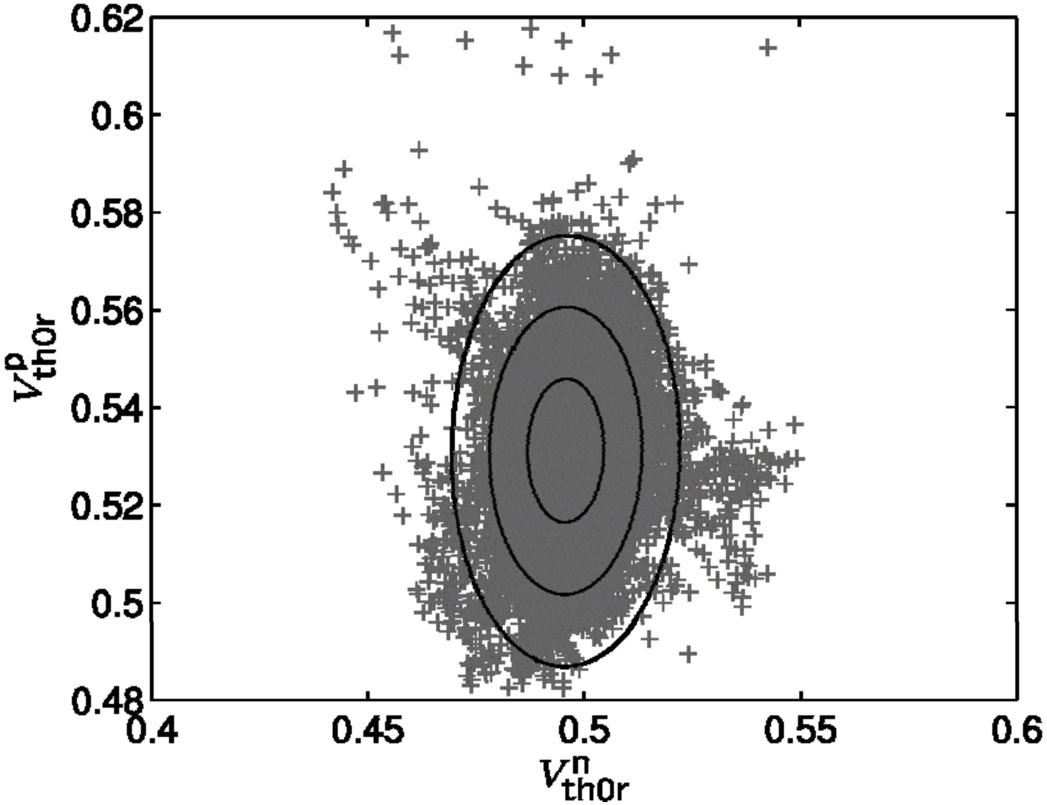
Wide/Short Threshold Voltage PDFs



dashed: MC from BPV Model

solid: Measurements

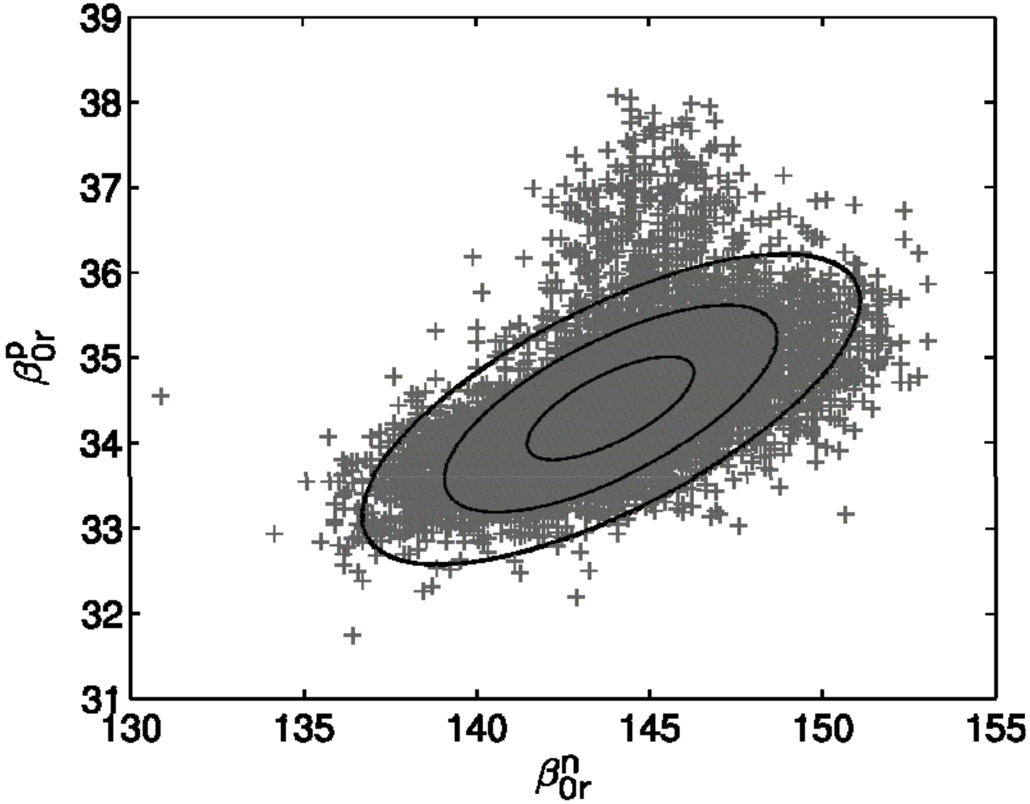
Wide/Long Threshold Voltage, P vs. N



solid lines: $\pm 1,2,3\sigma$ MC from BPV Model

+ Measurements

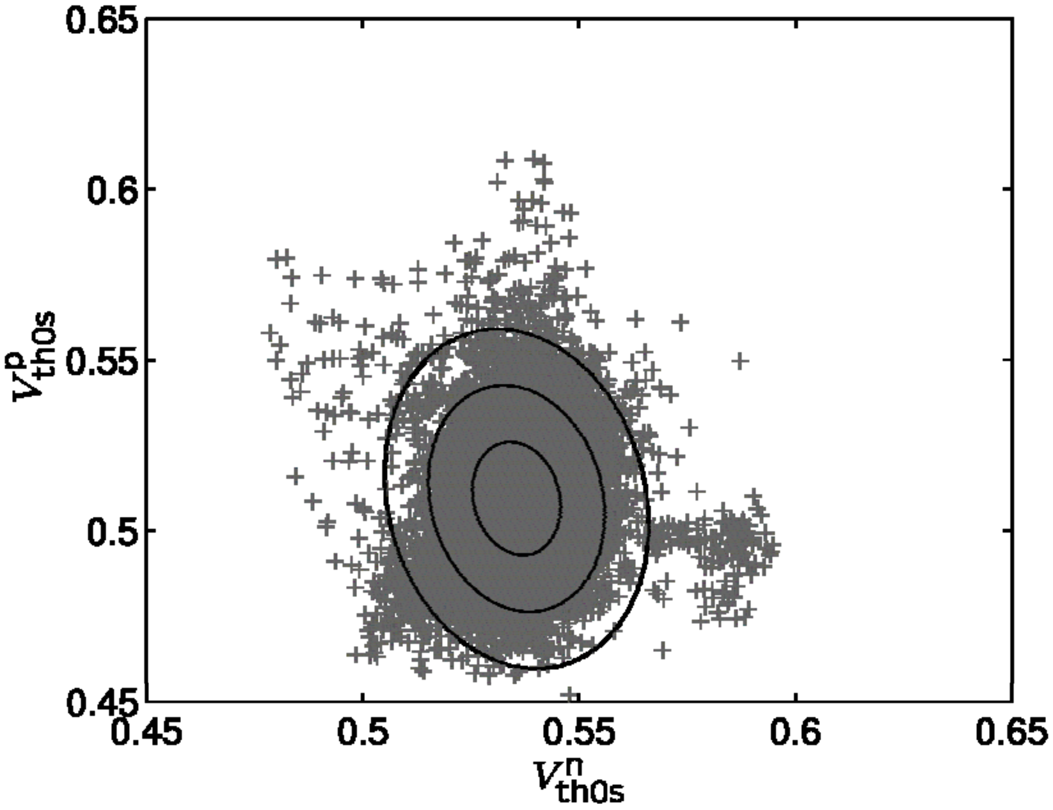
Wide/Long Gain Factor, P vs. N



solid lines: $\pm 1, 2, 3\sigma$ MC from BPV Model

+ Measurements

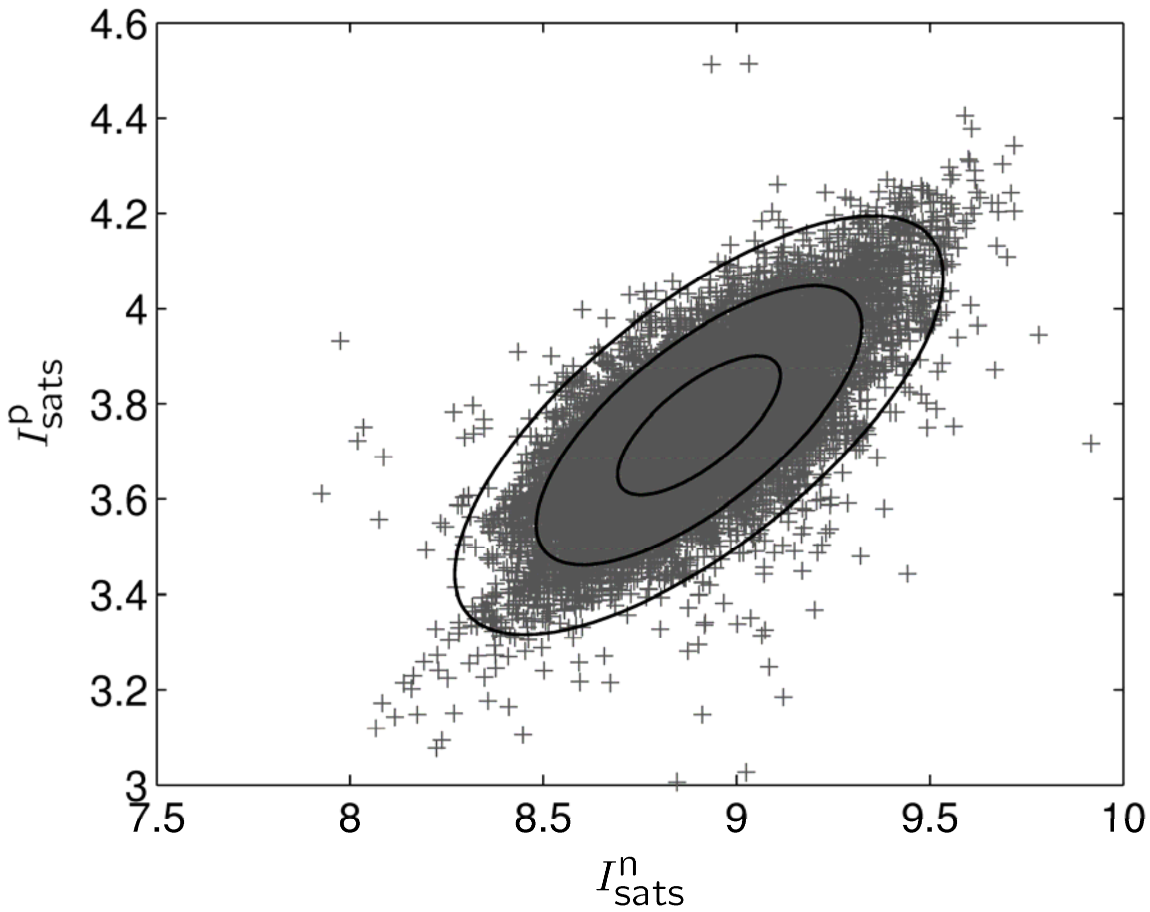
Wide/Short Threshold Voltage, P vs. N



solid lines: $\pm 1,2,3\sigma$ MC from BPV Model

+ Measurements

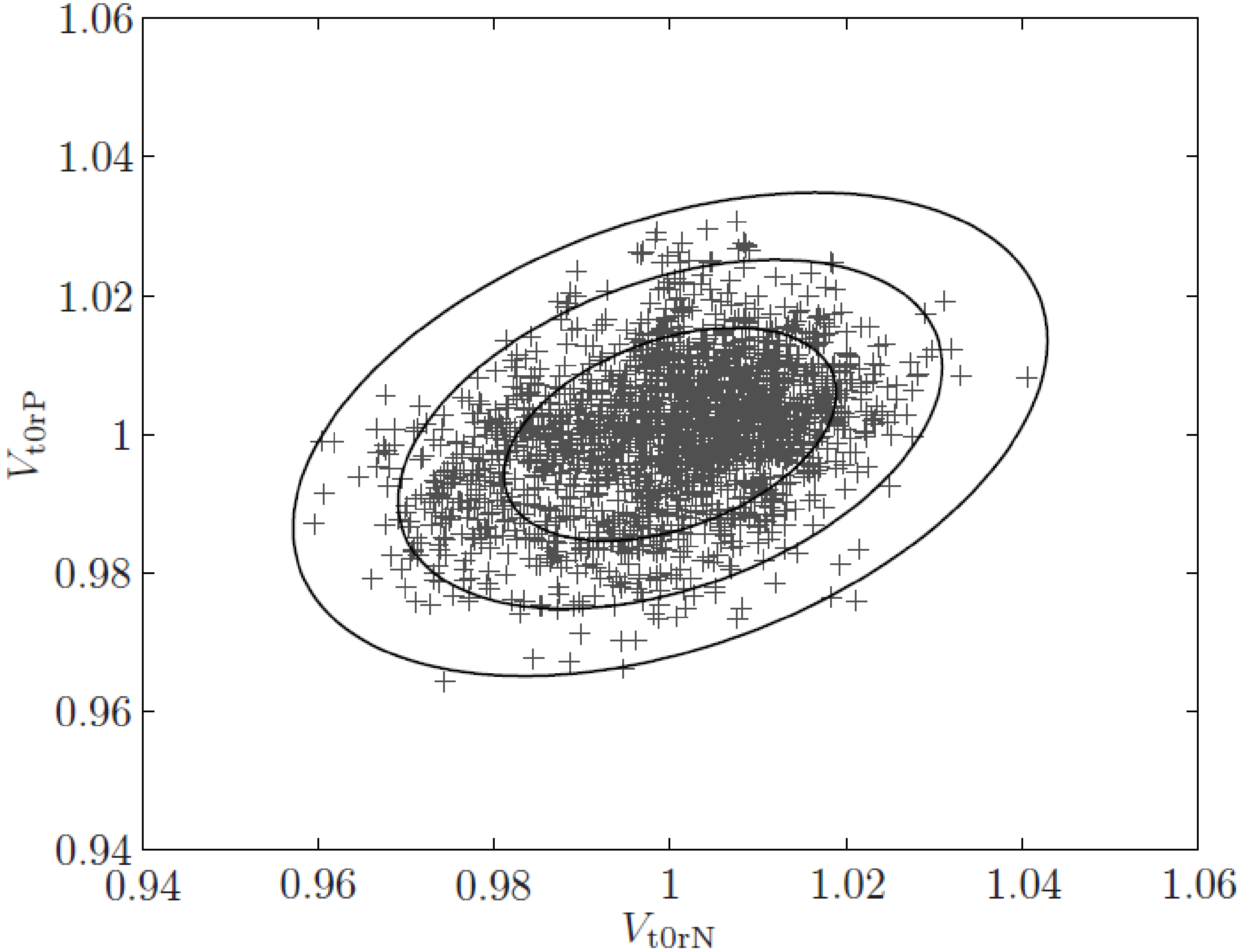
Wide/Short Saturated Drain Current, P vs. N



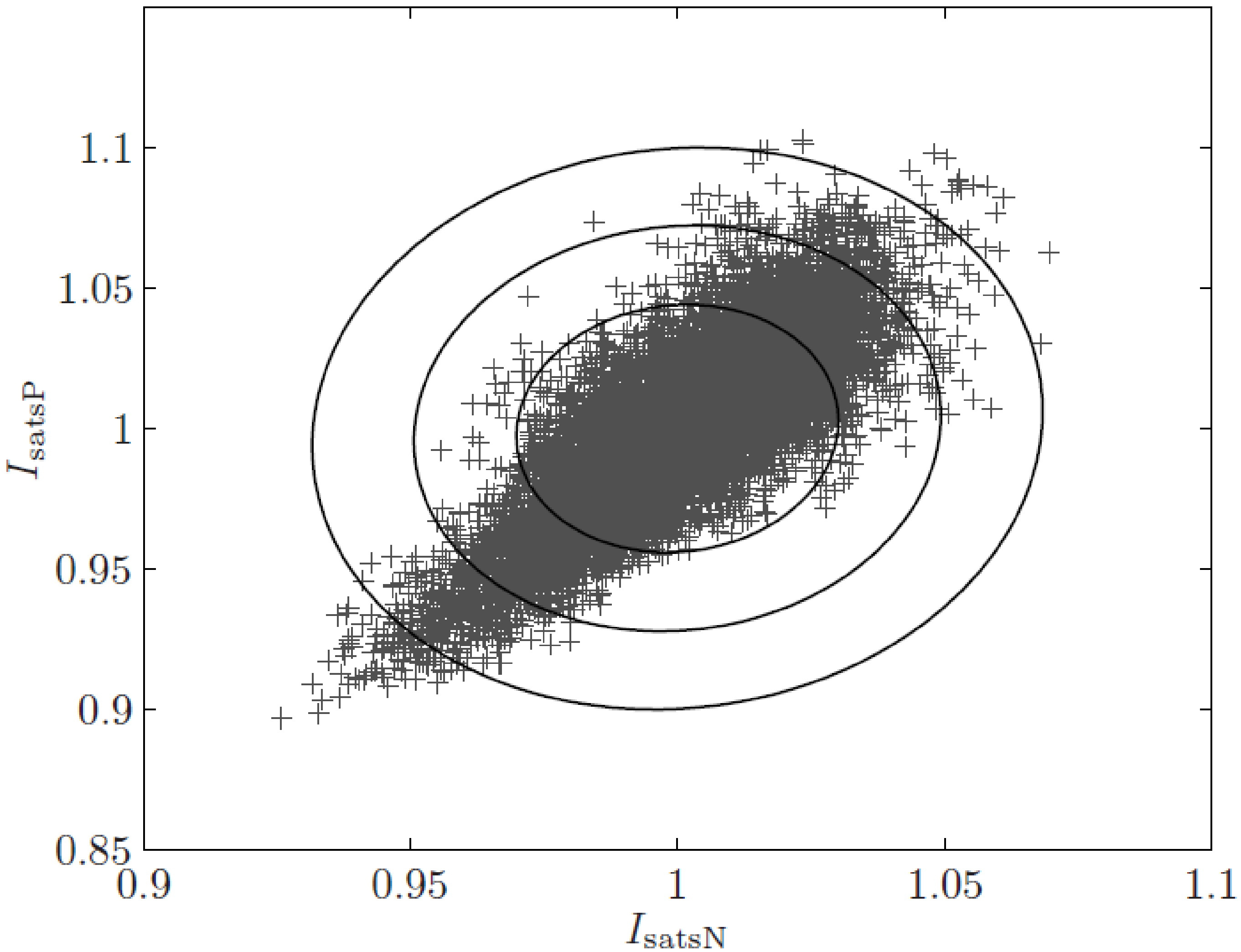
PSP Example

- Poly CD and out-diffusion length already available as separate parameters
  - no need to add as equations
  - poly CD parameter made common to PMOS and NMOS
- Threshold voltage depends on V_{fb}, t_{ox}, N_{sub}
- Body effect depends on t_{ox}, N_{sub}
- V_{fb}, N_{sub} are separate for PMOS and NMOS
- t_{ox} is common between PMOS and NMOS
- Include body effect and correlation between V_{t0} of PMOS and NMOS as fitting quantities
  - extends analytic correlation analysis to general numerical procedure
  - enables statistics of t_{ox} to be determined from only dc data

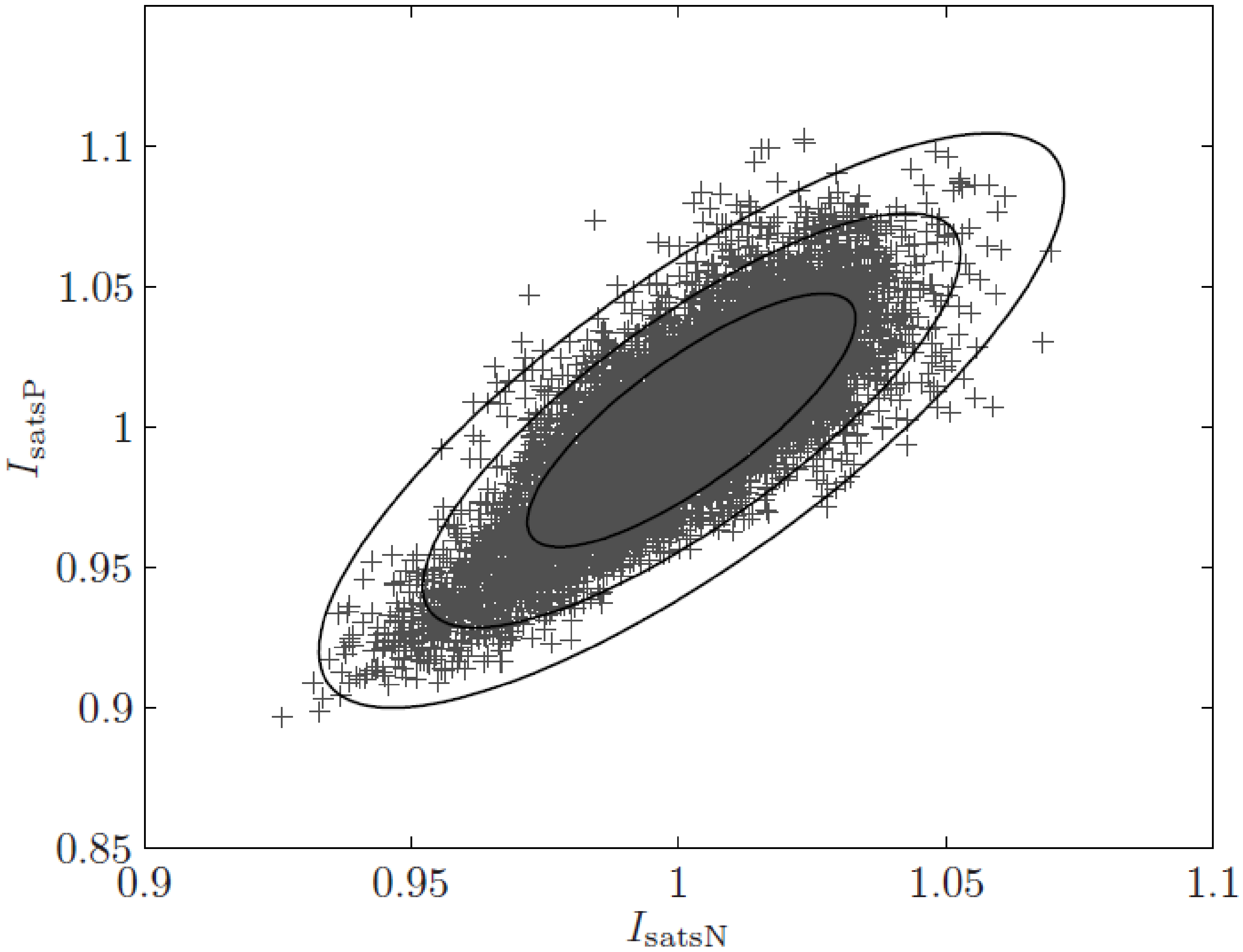
Fitting Correlation Enables Extraction of t_{ox} Variation



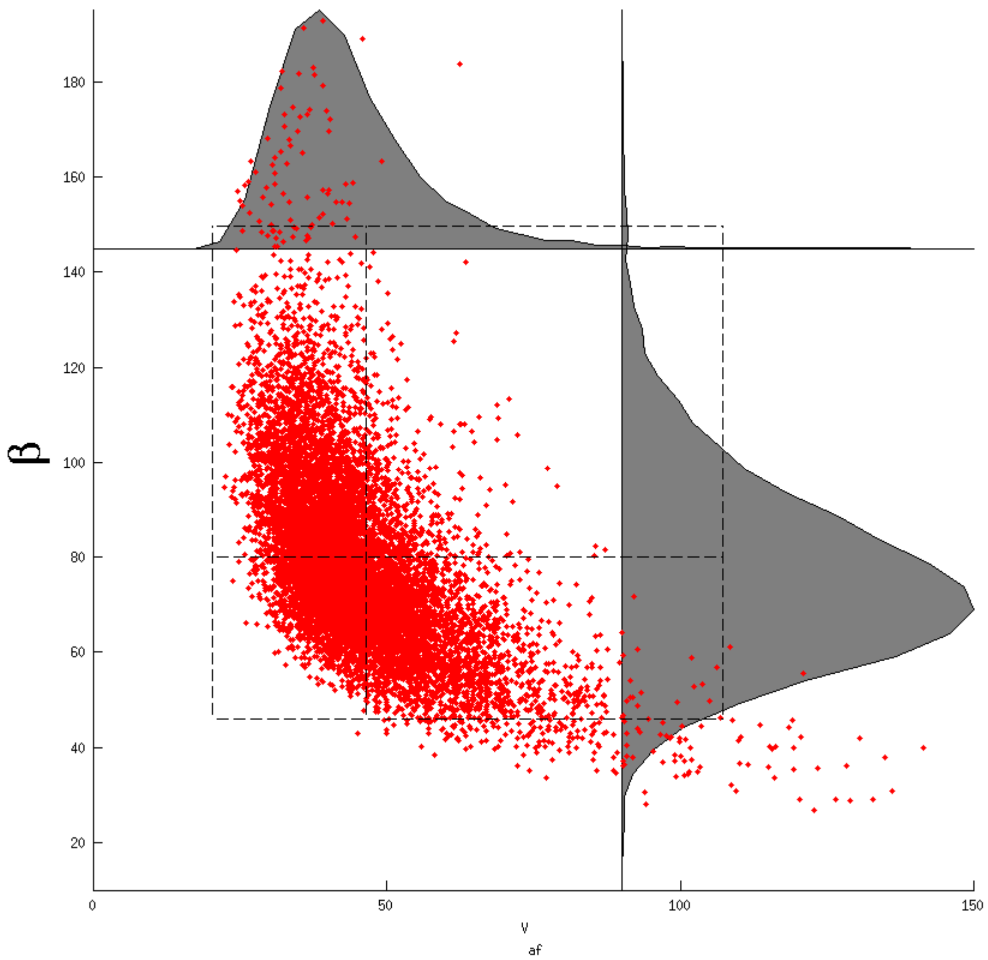
Without Explicit Fitting of I_{dsat} Correlation



With Explicit Fitting of I_{dsat} Correlation

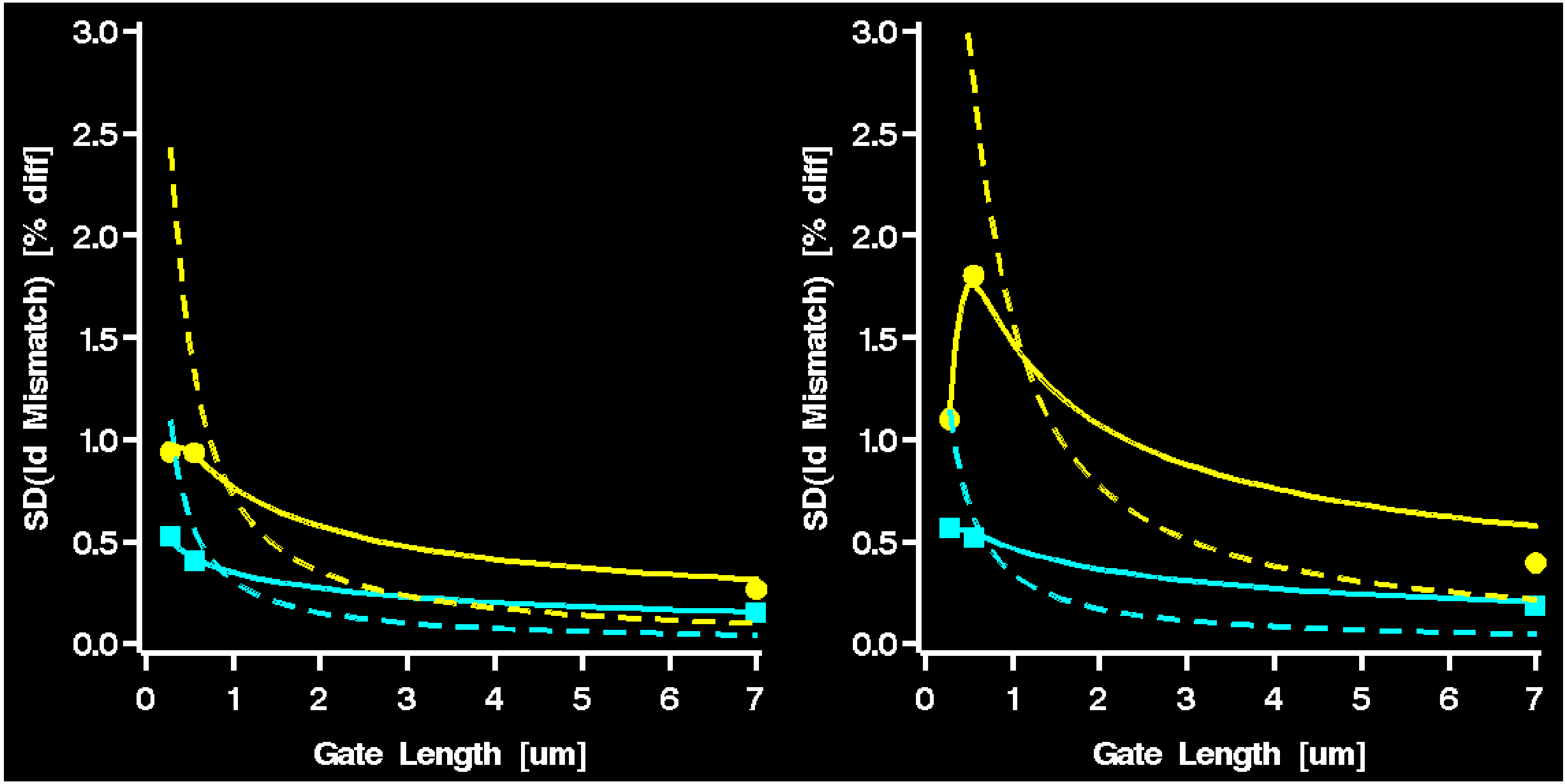


Same Formalism Applies to BJTs

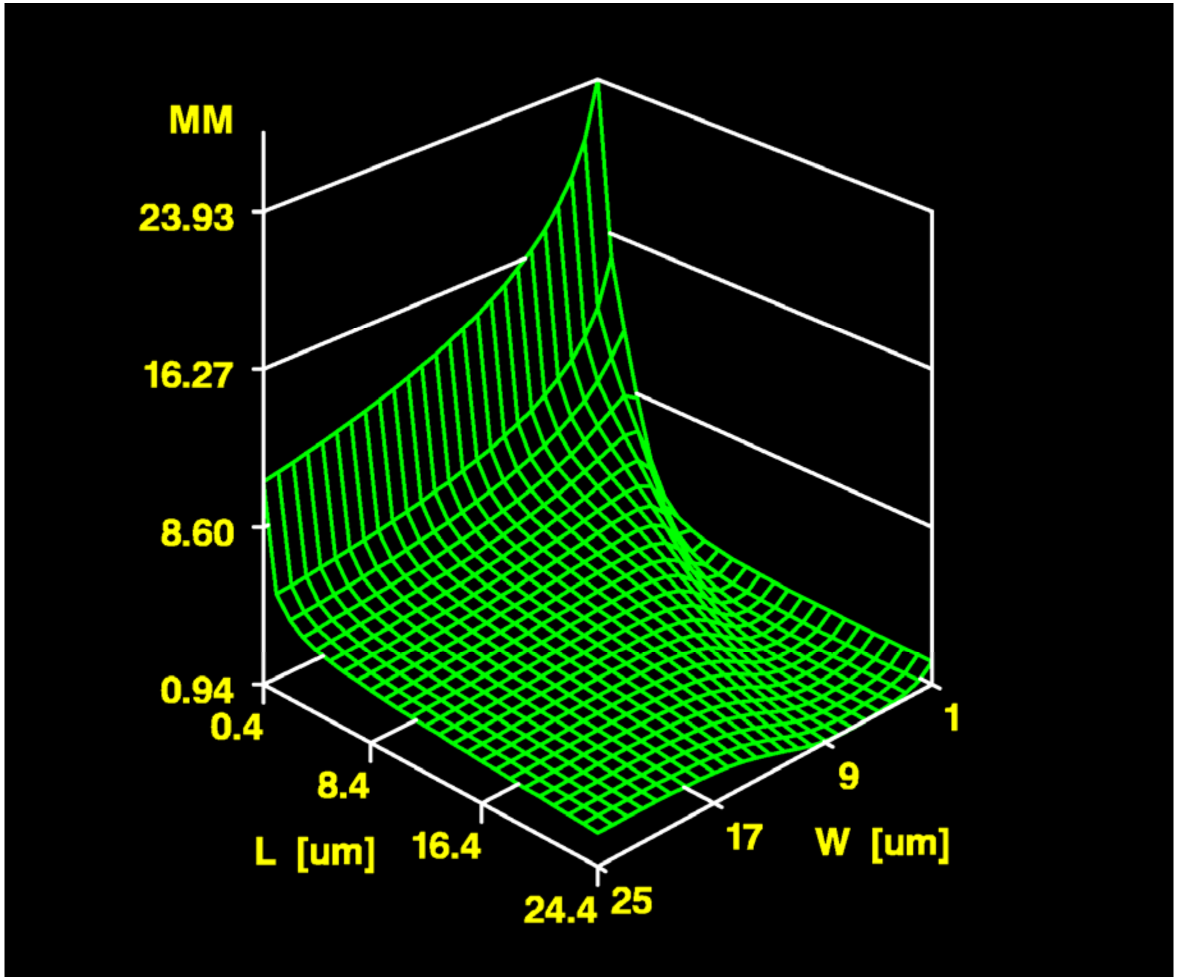


Same Formalism Applies to Mismatch

V_{bs}=0 V_{bs}=-2.5



Current Mirror Mismatch, 10μA



Same Formalism Applies to e_m from Circuits

- Goal is accurate modeling of variation of e_m of circuits
- In general e_m are from devices
  - models assumed to be reasonable
  - if device variations are modeled well, expectation is circuit variations will also be modeled well
- There is nothing in the BPV formalism that precludes some or all e_m from being circuit performances
- Added ring oscillator (RO) variations to the procedure
  - measured at same sites as dc data were measured
- Depends on both PMOS and NMOS devices

BPV Circuit Application: PSP for 0.18μm CMOS

Device (W/L, μm)	e_m	Description
Large (10/10)	V_{tr}	Threshold voltage for large device
Short (10/0.2)	V_{ts}	Threshold voltage for short device
	I_{ds}	Saturation current for short device
Narrow (0.24/10)	V_{tn}	Threshold voltage for narrow device
Small (0.24/0.2)	V_{tm}	Threshold voltage for small device
	I_{dm}	Saturation current for small device
RO	t_d	Gate delay for ring oscillators

p_i	Description
TOXO	Oxide thickness
VFBO	Geometry-independent flatband voltage
VFBL	Length-dependent flatband voltage
UO	Zero-field mobility
LAP	Channel length variation
WOT	Channel width variation

BPV Sensitivity Matrix Structure (t_{ox} is FPV)

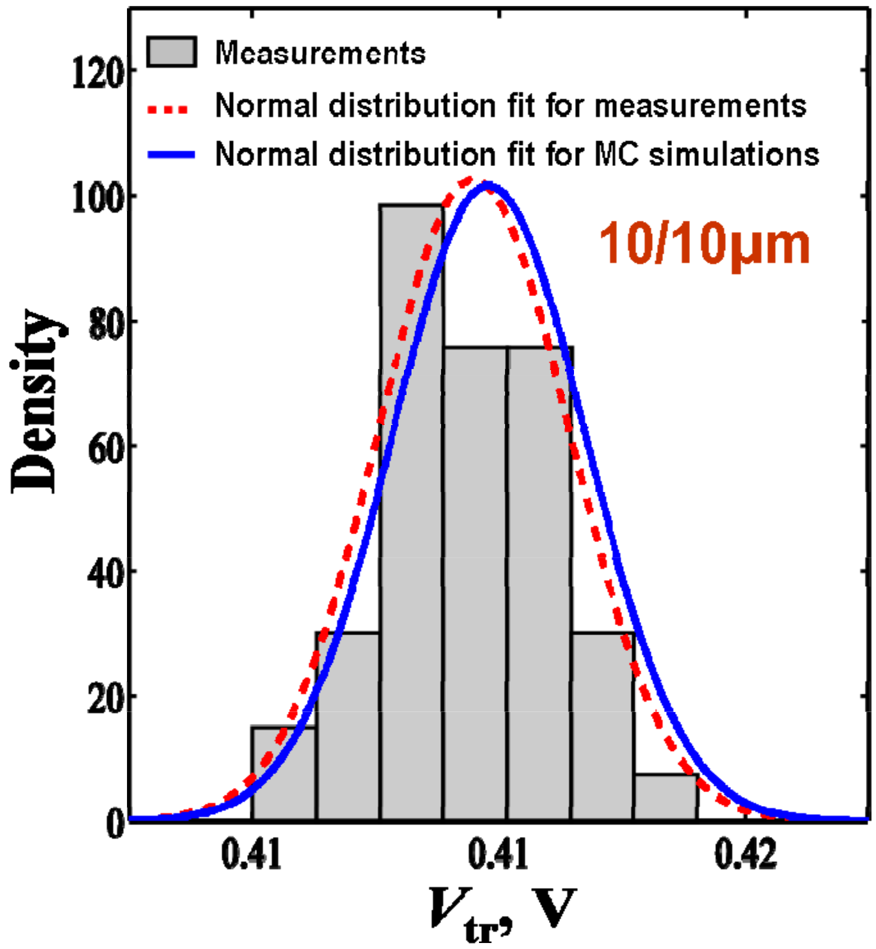
NMOS

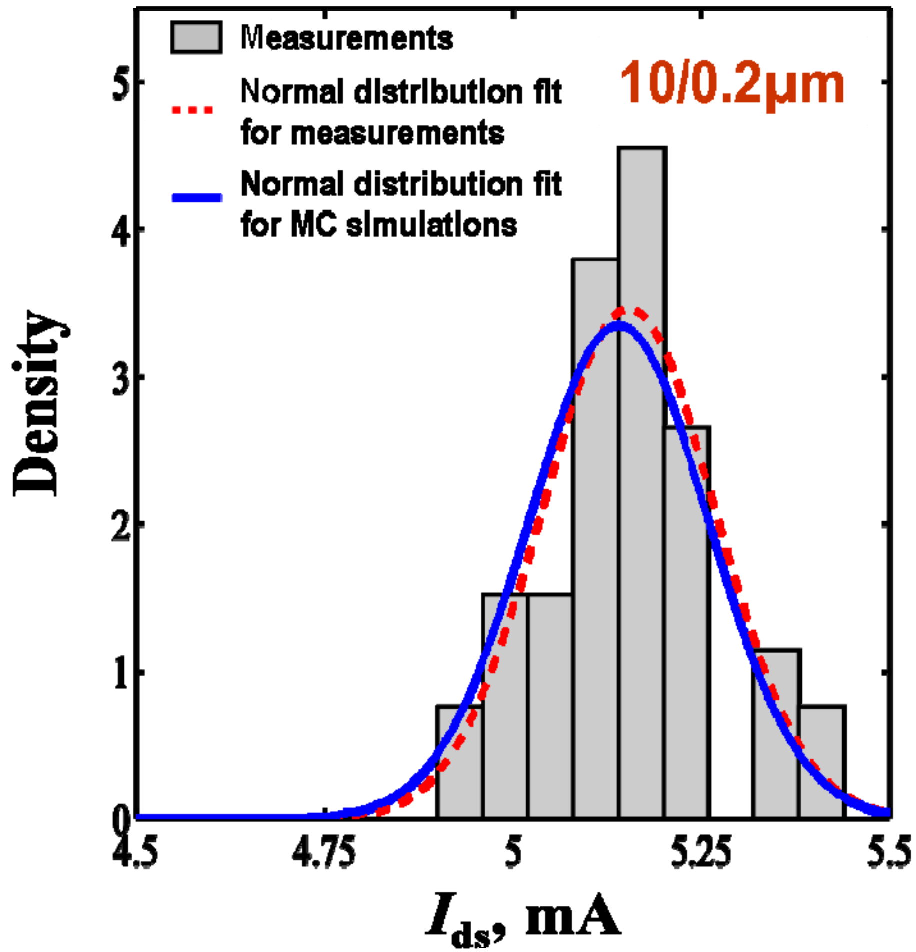
PMOS

circuit

$$\begin{bmatrix} \sigma^2_{\boldsymbol{e}_N} - \left(t_{ox}\dfrac{\partial \boldsymbol{e}_N}{\partial t_{ox}}\right)^2 \sigma^2_{\frac{\delta t_{ox}}{t_{ox}}} \\ \sigma^2_{\boldsymbol{e}_P} - \left(t_{ox}\dfrac{\partial \boldsymbol{e}_P}{\partial t_{ox}}\right)^2 \sigma^2_{\frac{\delta t_{ox}}{t_{ox}}} \\ \sigma^2_{\boldsymbol{e}_C} - \left(t_{ox}\dfrac{\partial \boldsymbol{e}_C}{\partial t_{ox}}\right)^2 \sigma^2_{\frac{\delta t_{ox}}{t_{ox}}} \end{bmatrix} = \begin{bmatrix} \left(\dfrac{\partial \boldsymbol{e}_N}{\partial \boldsymbol{p}_N}\right)^2 & 0 \\ 0 & \left(\dfrac{\partial \boldsymbol{e}_P}{\partial \boldsymbol{p}_P}\right)^2 \\ \left(\dfrac{\partial \boldsymbol{e}_C}{\partial \boldsymbol{p}_N}\right)^2 & \left(\dfrac{\partial \boldsymbol{e}_C}{\partial \boldsymbol{p}_P}\right)^2 \end{bmatrix} \begin{bmatrix} \sigma^2_{\boldsymbol{p}_N} \\ \sigma^2_{\boldsymbol{p}_P} \end{bmatrix}$$

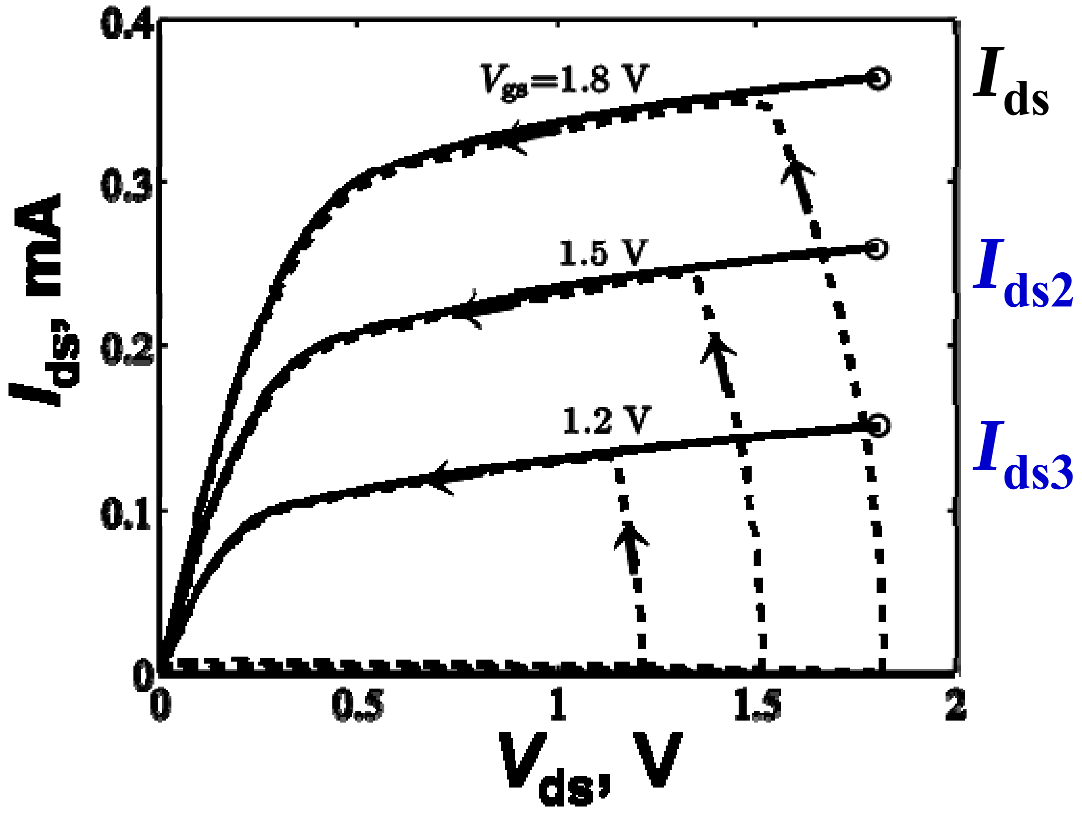
MC Based on BPV Model



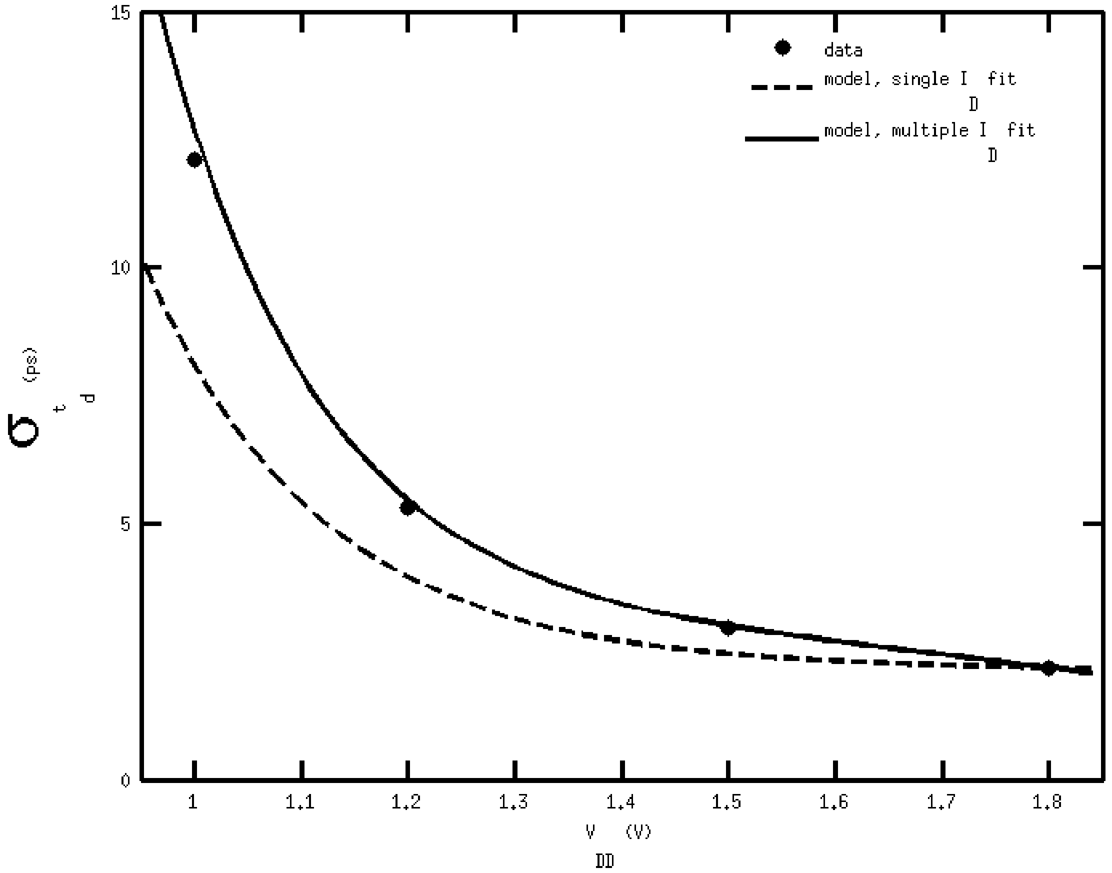


Addition of Extra e_m

- Fitting I_{ds} only did not lead to best fit of statistics of RO delay over V_{dd}
- Added drain current at additional V_{gs} biases as targets



Improved Modeling of RO Delay Variability over V_{dd}



Issues and Opportunities

- Local variation is inherent in all small devices
  - for "nominal" device characterization need median data
    - > otherwise $V_{t0}(L)$ can be modeling "noise"
    - > cannot average data in weak and moderate inversion
- In the past, $\sigma_{mm} << \sigma_{gl}$ so mismatch characterization could be done independently from global variation characterization
  - no longer the case
  - need to "correct" fab data: $\sigma_{gl}^2 = \sigma_{fab}^2 - \sigma_{mm}^2$
- Sensitivity-based corner model generation
- Education of limitations of corner models
  - mixing global and local variation properly
- Statistical simulation needs to better leverage sensitivities
  - these can be calculated automatically from Verilog-A code

Summary

- BPV is cheating
  - fudges parameter statistics to force-fit device variations
  - choose targets wisely and this is exactly what is needed for design
- BPV works for both global and local statistical variations
  - targets differ, procedure does not
- Recent extensions to BPV
  - include CMOS circuit performances as targets
    - > couples NMOS and PMOS statistical characterization
  - include more targets than minimum number
    - > gives improved modeling of $t_d(V_{dd})$
  - include skewness as a target
    - > allows fitting of nonlinear $\boldsymbol{e}_m(\boldsymbol{p}_i)$
  - include correlations as a target
    - > allows generic fitting of correlations with uncorrelated parameters

BPV References

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- BPV extension to incorporate correlation between electrical performances were primarily developed and implemented by Xin Li and Ivica Stevanović
- Some results presented here are from models generated by Ivica Stevanović, Xin Li, and Patrick Drennan