Fast classification using sparsely active spiking networks

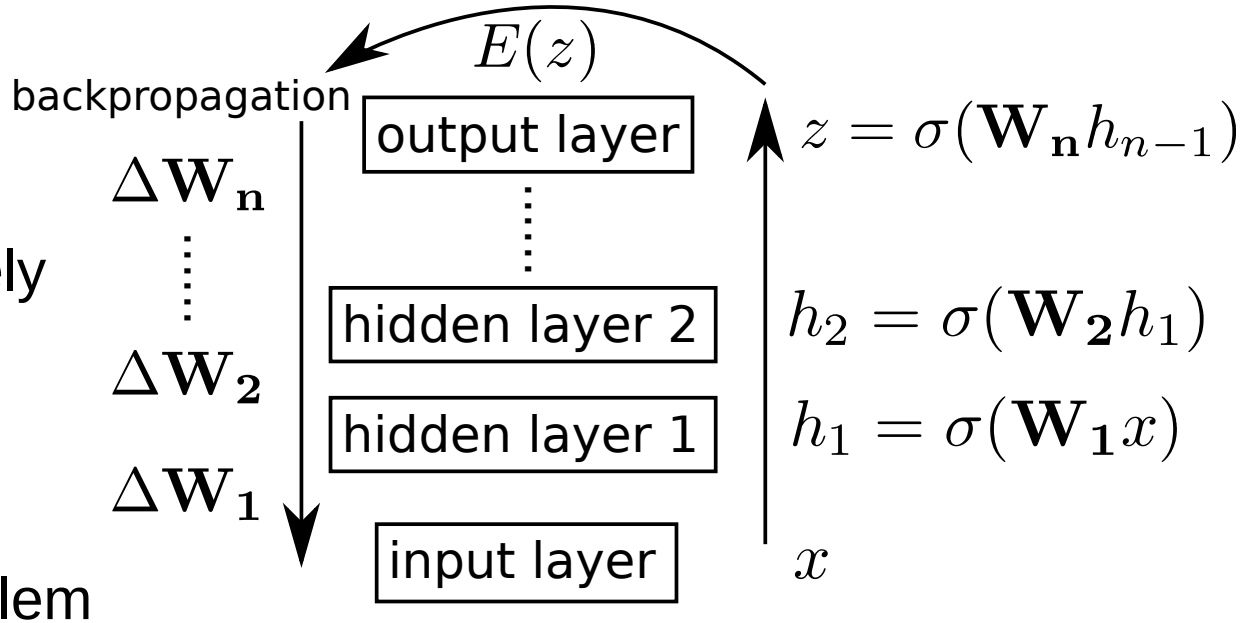
Hesham Mostafa
Institute of neural computation, UCSD

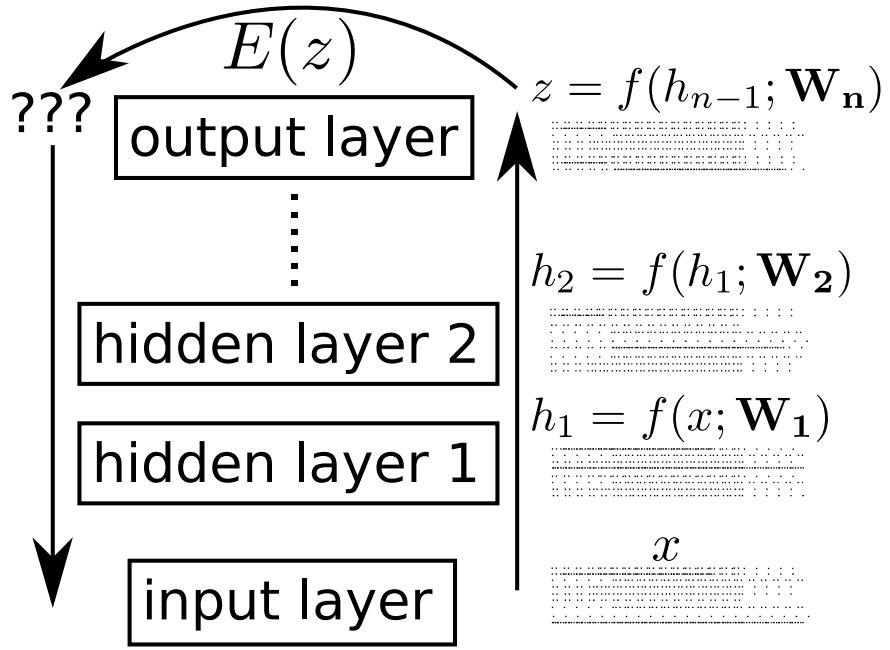
Artificial networks vs. spiking networks

Multi-layer networks are extremely powerful function approximators.

Backpropagation is the most effective method we know of to solve the credit assignment problem in deep artificial networks

<u>How do we solve the credit assignment problem in multi-layer spiking networks?</u>

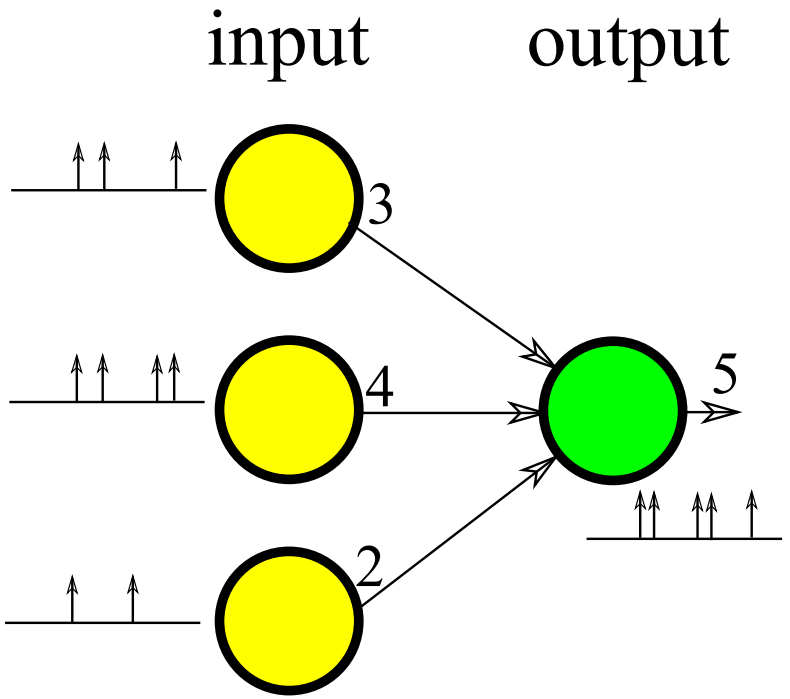




Neural codes and gradient descent

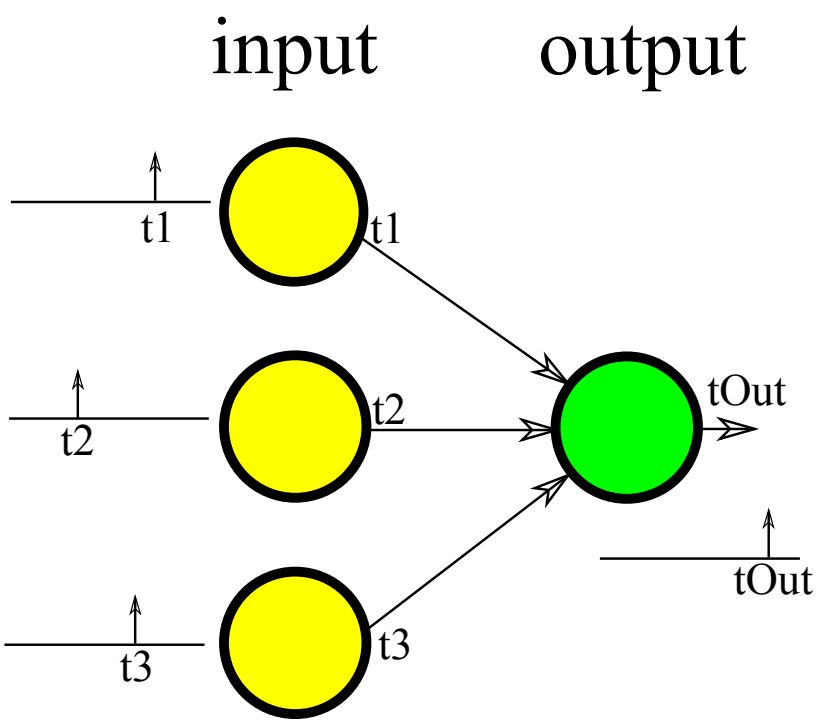
Rate coding:

- Spike counts/rates are discrete quantities
- Gradient is zero almost everywhere
- Only indirect or approximate gradient descent training possible



Temporal coding

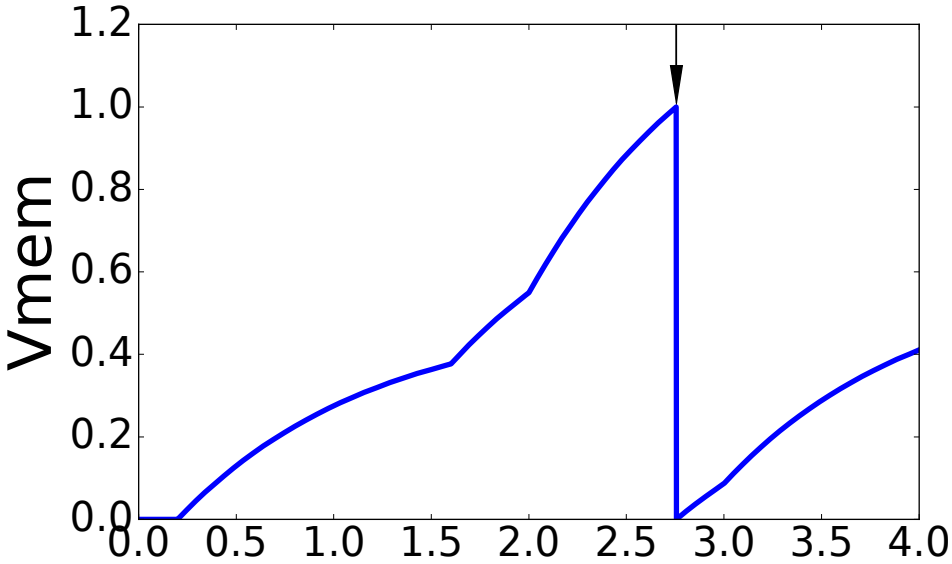
- Spike times are analog quantities
- Gradient of output spike time w.r.t input spike times is well-defined and non-zero
- Direct gradient descent training possible



The neuron model

Non-leaky integrate and fire neuron

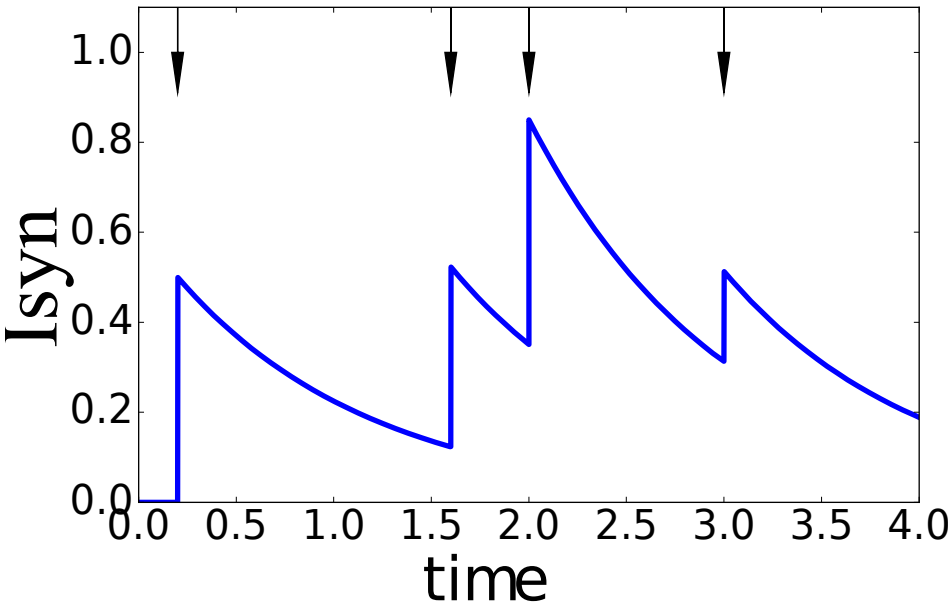
$$\frac{dVmem(t)}{dt} = Isyn(t)$$ (firing threshold is 1)



Exponentially decaying synaptic current

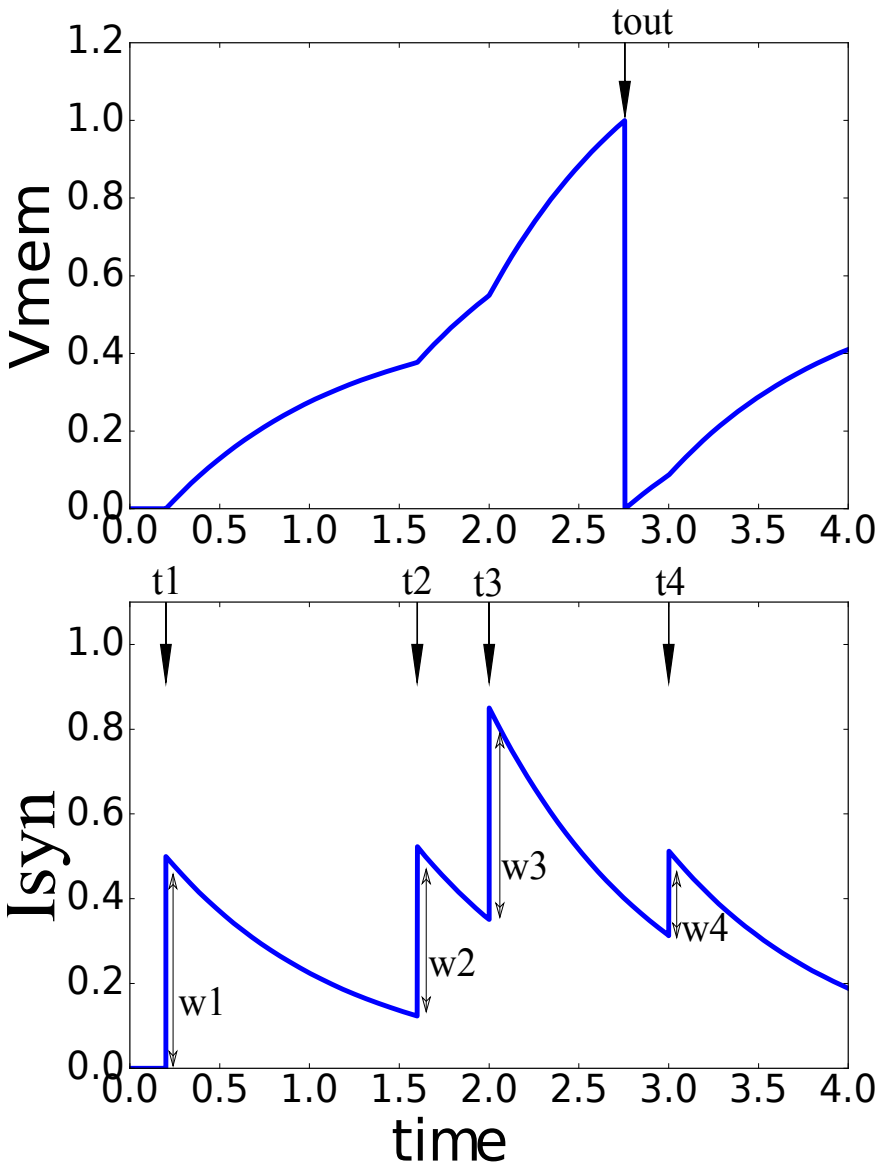
$$Isyn(t) = \sum_i w_i \exp(-(t - t_i))\Theta(t - t_i)$$

$\Theta(t - t_i)$: Step function



The neuron's transfer function

$$exp(t_{out}) = \frac{w_1 exp(t_1) + w_2 exp(t_2) + w_3 exp(t_3)}{w_1 + w_2 + w_3 - 1}$$



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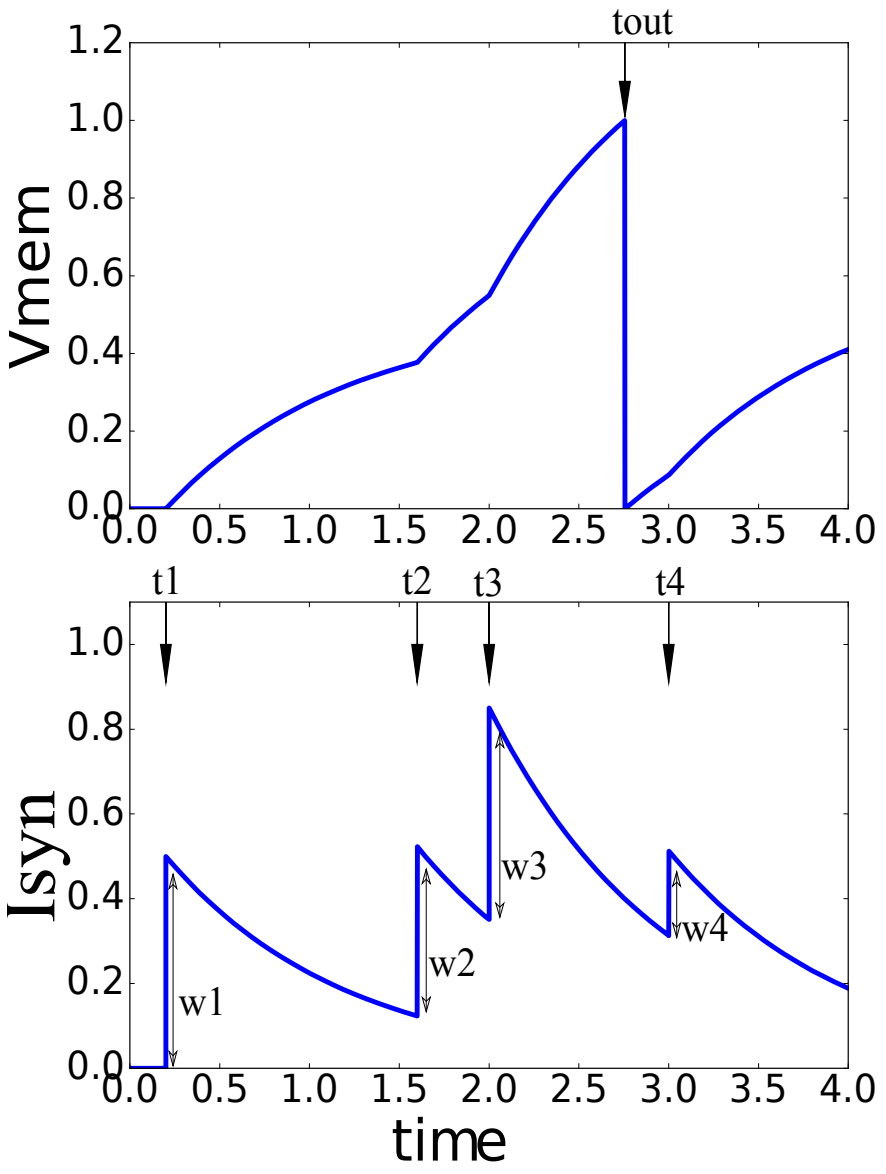
In general:

$$exp(t_{out}) = \frac{\sum_{i \in C} w_i exp(t_i)}{\sum_{i \in C} w_i - 1}$$

Where C is the causal set of input spikes (input spikes that arrive before output spike)



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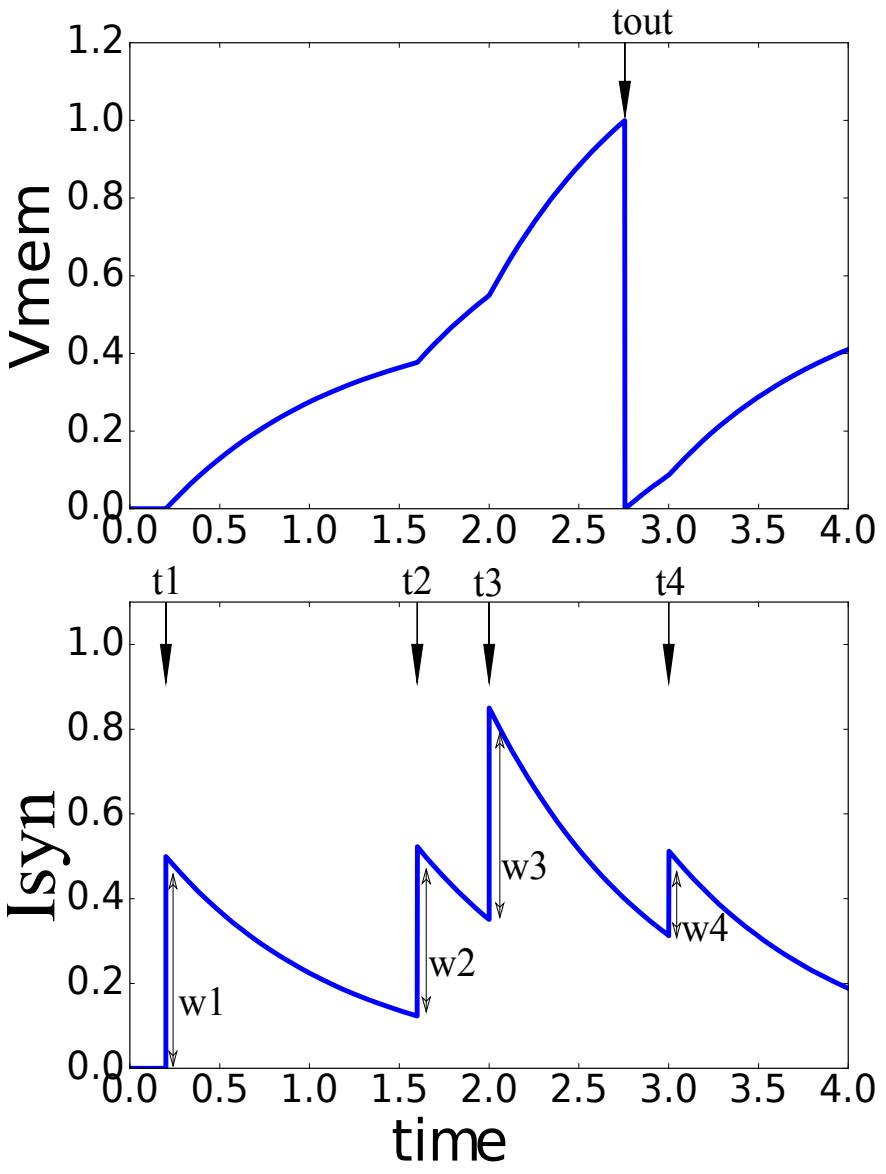
Where C is the causal set of input spikes (input spikes that arrive before output spike)

Time of the L^{th} output spike:

$$exp(t^L_{out}) = \frac{\sum_{i \in C} w_i exp(t_i)}{\sum_{i \in C} w_i - L}$$



Change of variables

$$exp(t_x) \rightarrow z_x$$

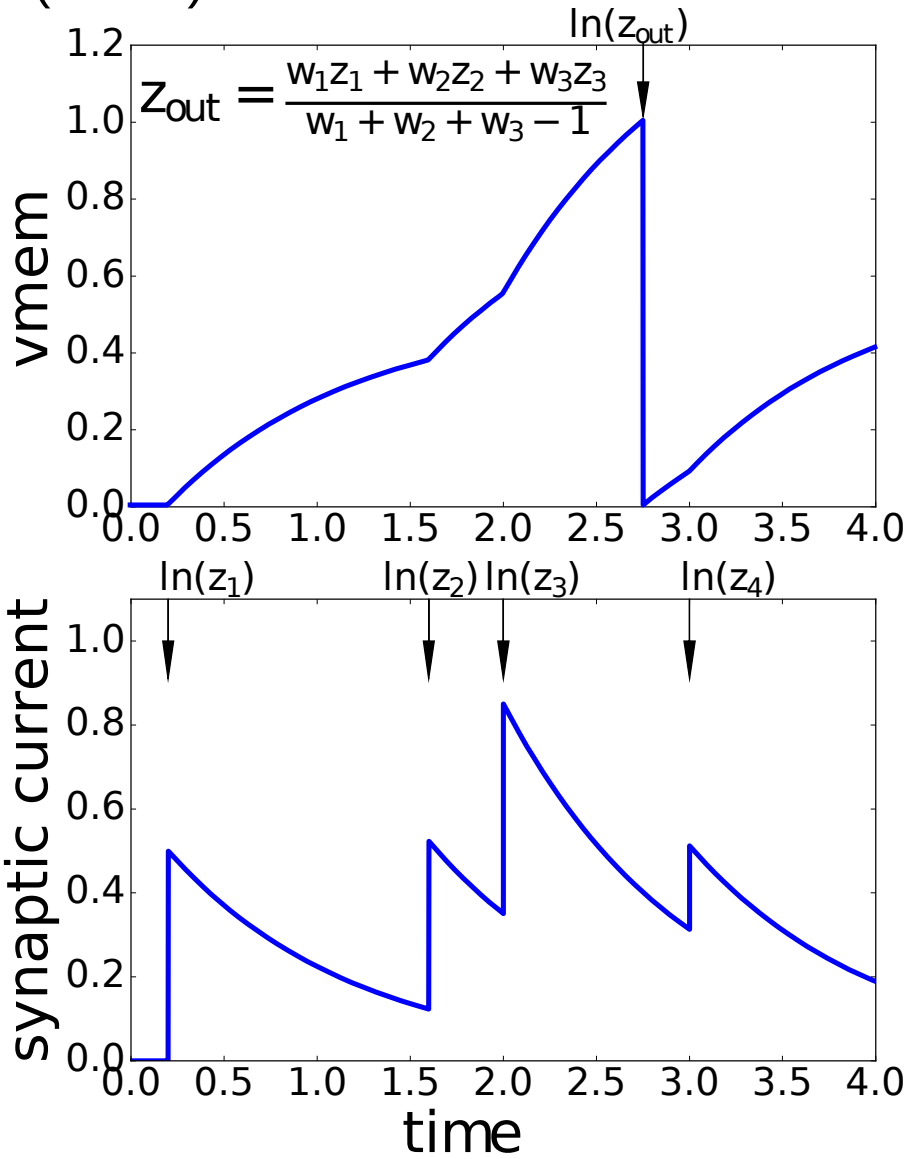
The neuron's transfer function then becomes piece-wise linear in the inputs (but not the weights):

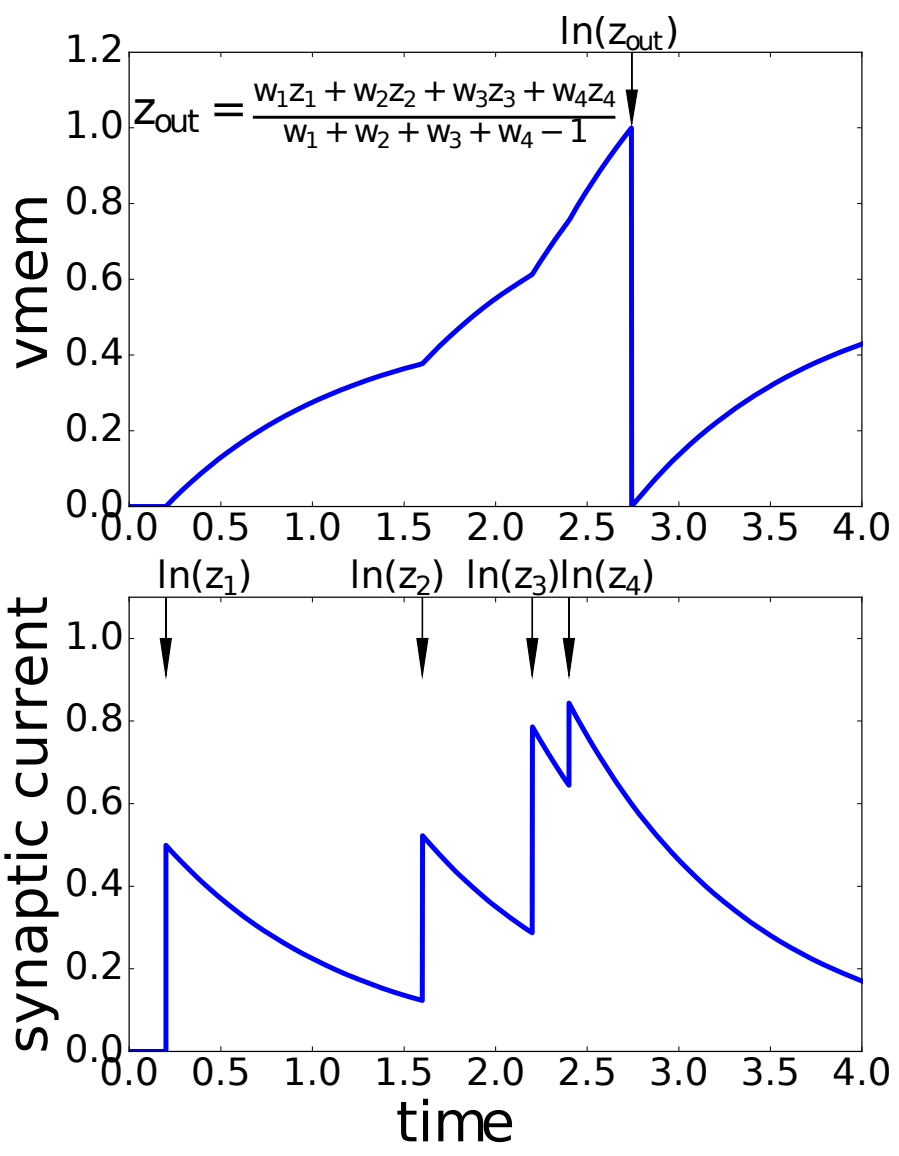
$$z_{out} = \frac{\sum_{i \in C} w_i z_i}{\sum_{i \in C} w_i - 1}$$

Where is the non-linearity?

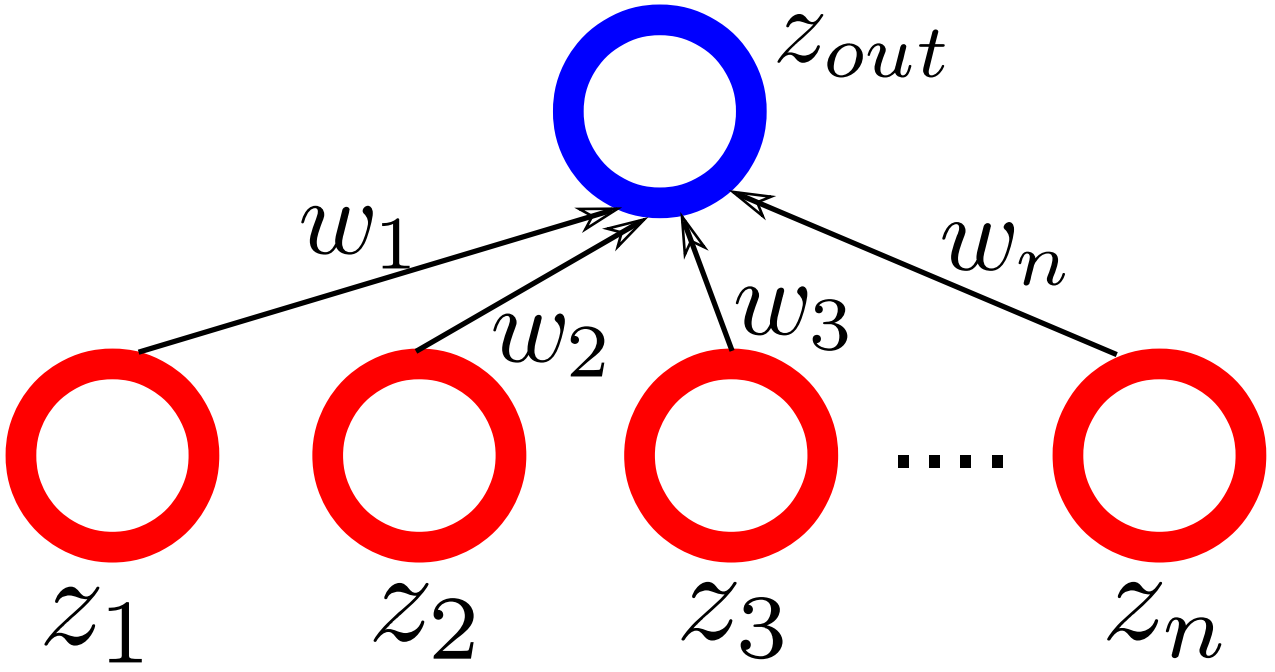
Non-linearity arises due to the input dependence of the causal set of input spikes

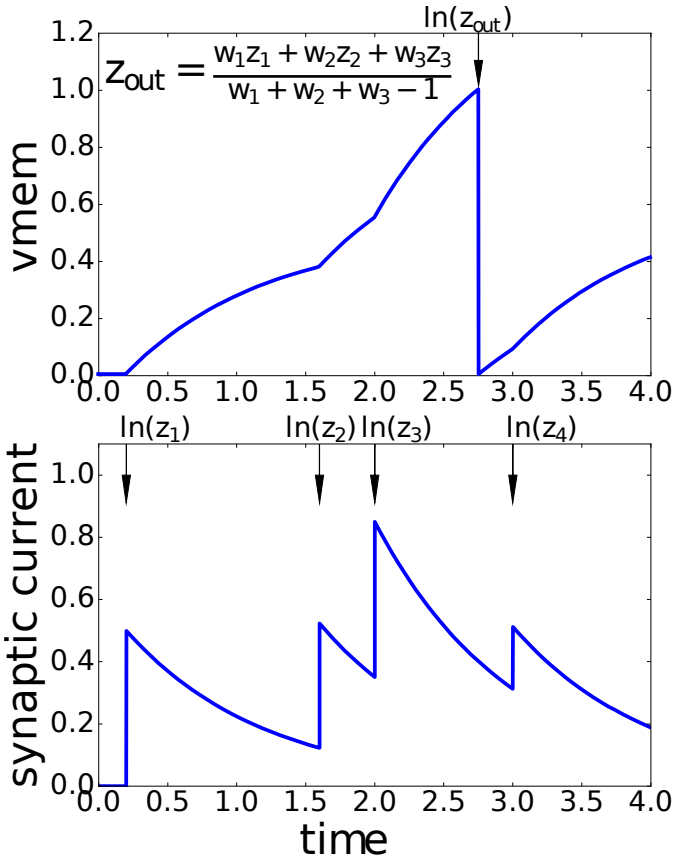
The piecewise linear input-output relation is reminiscent of Rectified Linear Units (ReLU) networks





What is the form of computation implemented by the temporal dynamics?





- To compute zout:
  - Sort {z1,z2,..,zn}
  - Find the causal set, C, by progressively considering more early spikes
  - Calculate $z_{out} = \sum_{i \in C} w_i z_i / (\sum_{i \in C} w_i - 1)$

Can not be reduced to the conventional ANN neuron: $z_{out} = f\left(\sum_i w_i z_i\right)$

Backpropagation

$$z_{out} = \frac{\sum_{i \in C} w_i z_i}{\sum_{i \in C} w_i - 1}$$

To use backpropagation to train a multi-layer network, we need the derivatives of the neuron's output w.r.t:

Weights

$$\frac{dz_{out}}{dw_p} = \begin{cases} \frac{z_p - z_{out}}{\sum_{i \in C} w_i - 1} & \text{if} \quad p \in C \\ 0 & \text{otherwise} \end{cases}$$

Inputs

$$\frac{dz_{out}}{dz_p} = \begin{cases} \frac{w_p}{\sum_{i \in C} w_i - 1} & \text{if} \quad p \in C \\ 0 & \text{otherwise} \end{cases}$$

Time of first spike encodes neuron's value. Each neuron is allowed to spike only once in response to an input pattern:

- Forces sparse activity. Training has to make maximum use of each spike
- Allows quick classification response

Classification Tasks

- We can relate the time of any spike differentiably to the times of all spikes that caused it
- We can impose any differentiable cost function on the spike times of the output layer and use backpropagation to minimize cost across training set
- In a classification setting, use a loss function that encourages the output neuron representing the correct class to spike first
- Since we have an analytical input-output relation for each neuron, training can be done using conventional machine learning packages (Theano/Tensorflow)

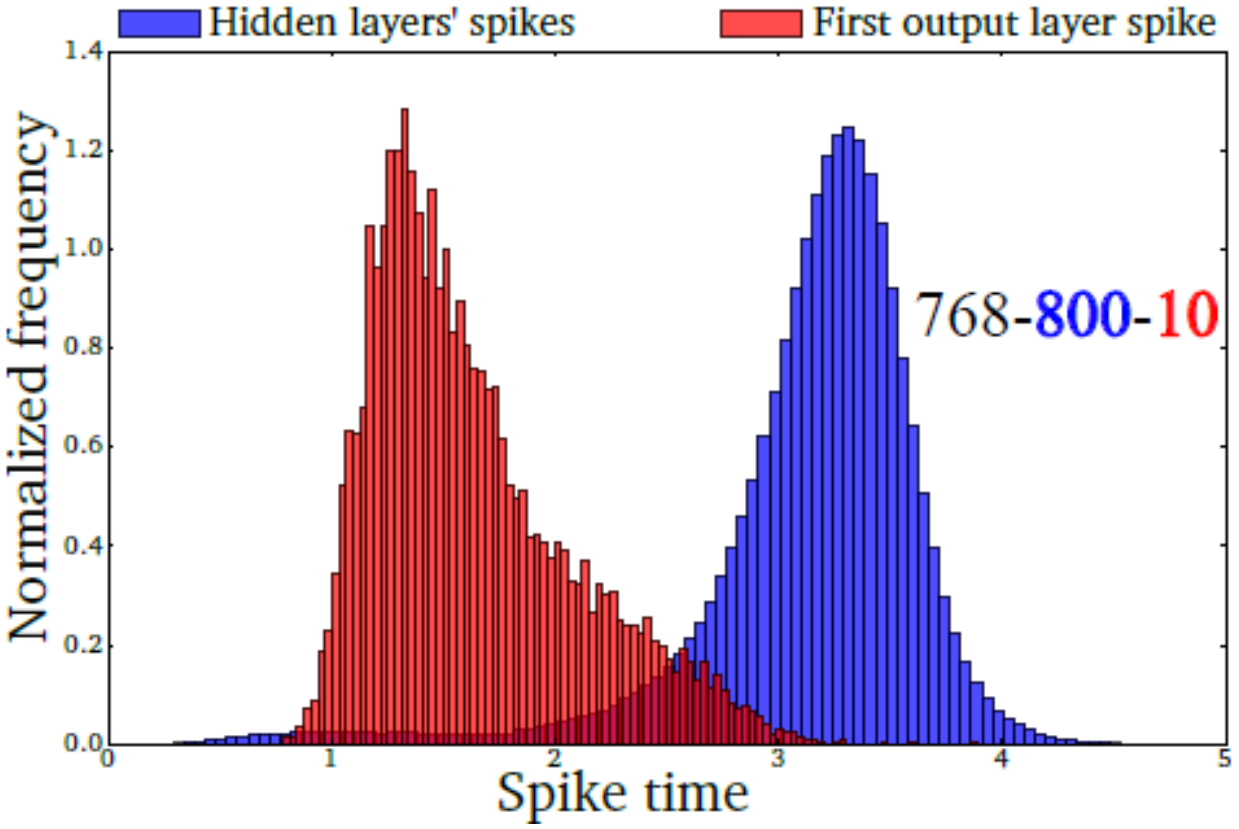
MNIST task

- Pixel values were binarized.
- High intensity pixels spike early
- Low intensity pixels spike late

Table 1: Performance results for the permutation-invariant MNIST task

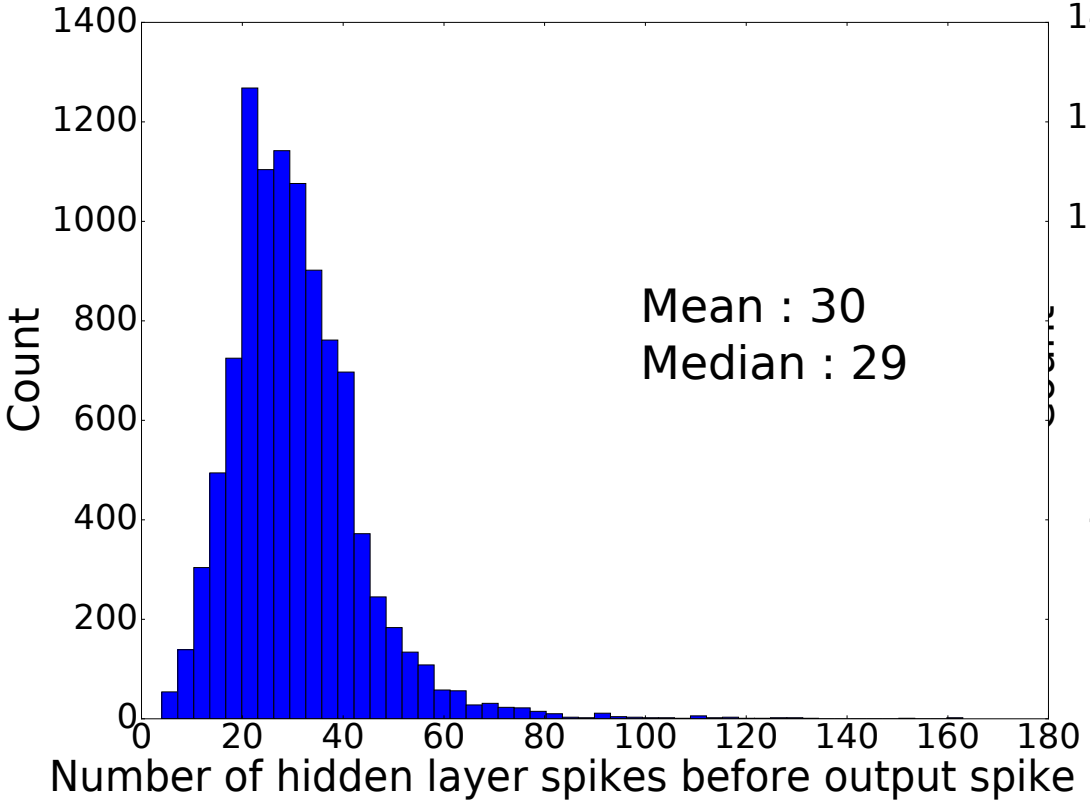
Network	Training set error	Test set error
784-800-10 (non-noisy training input)	0.013%	2.8%
784-800-10 (noisy training input)	0.005%	2.45%

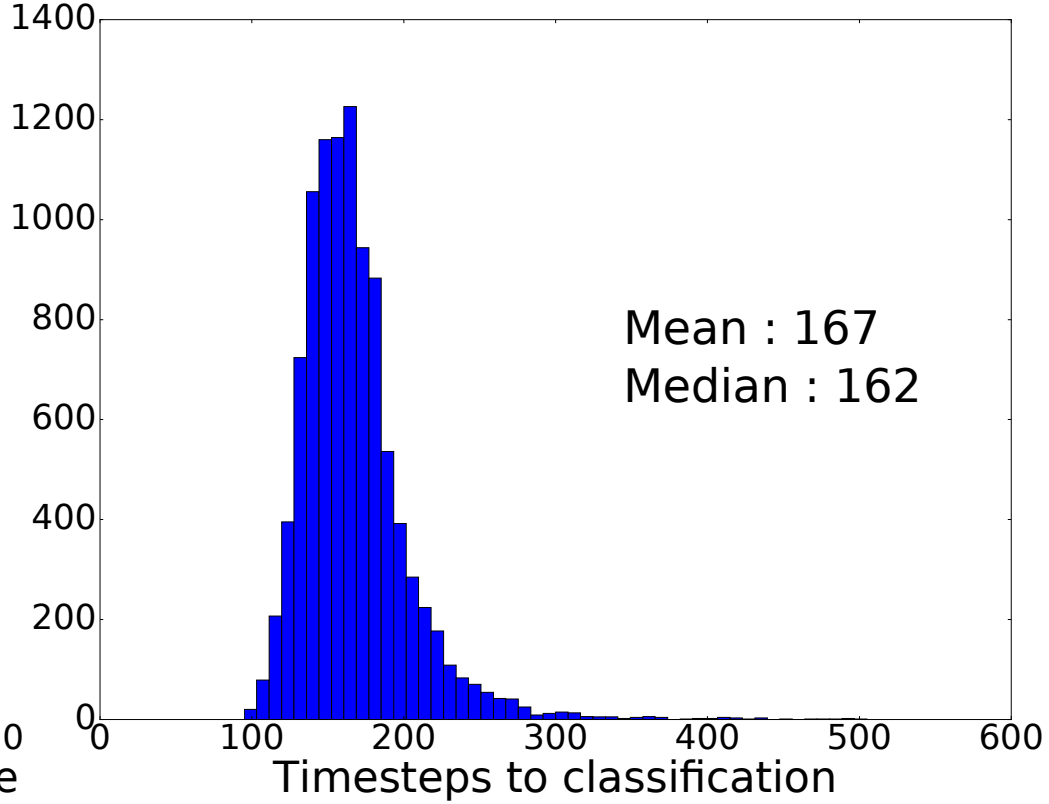
Classification is extremely rapid



- A decision is made when the first output neuron spikes
- A decision is made after only 25 spikes (on average) from the hidden layer in the 768-800-10 network, i.e, only 3% of the hidden layer neurons contribute to each classification decision

FPGA prototype





- 97% test set classification accuracy on MNIST in a 784-600-10 network (8-bit weights)
- Average number of spikes until classification: 139
- Only 13% of input to hidden weights are looked up
- Only 5% of hidden to output weights are looked up

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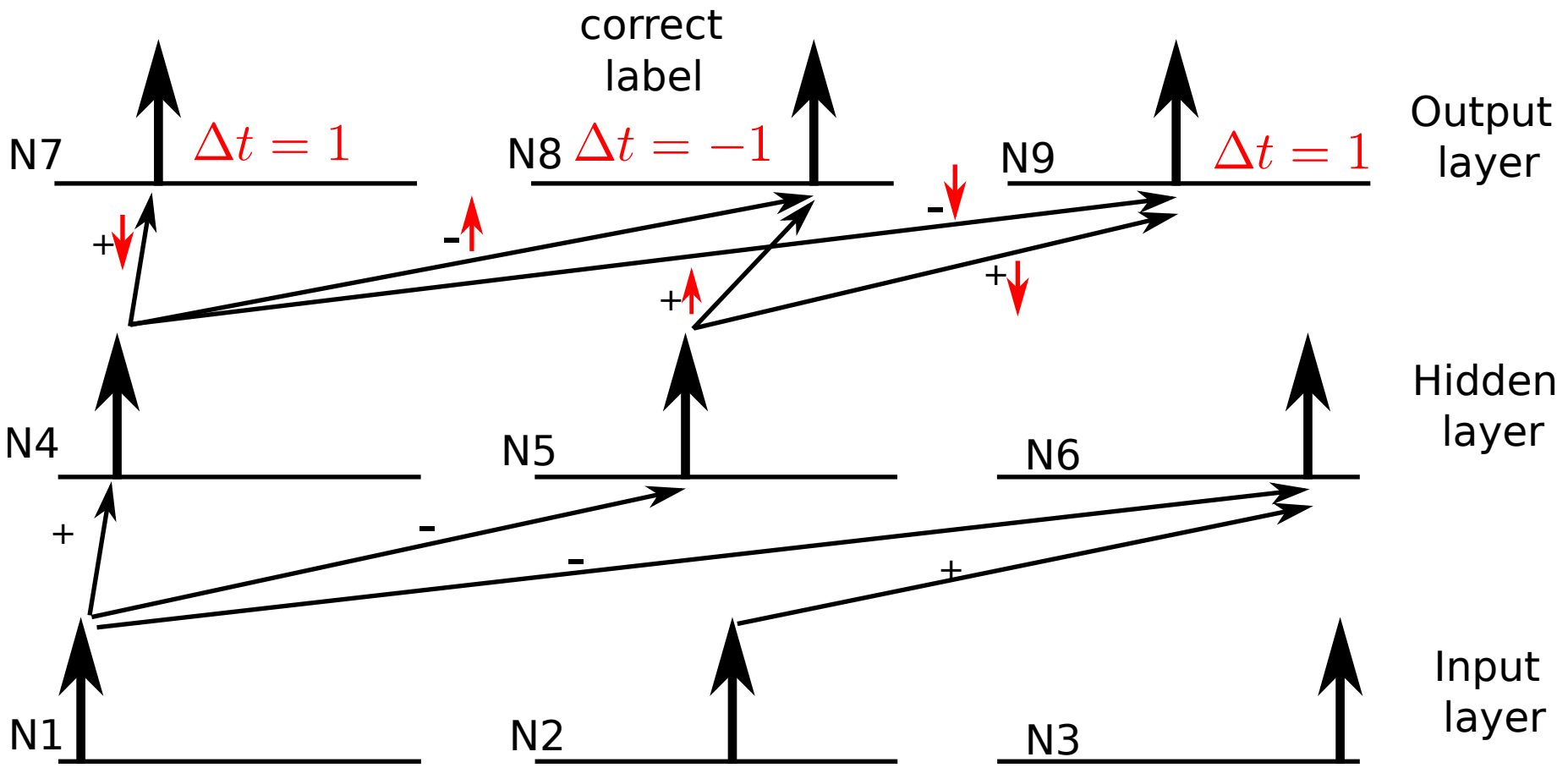
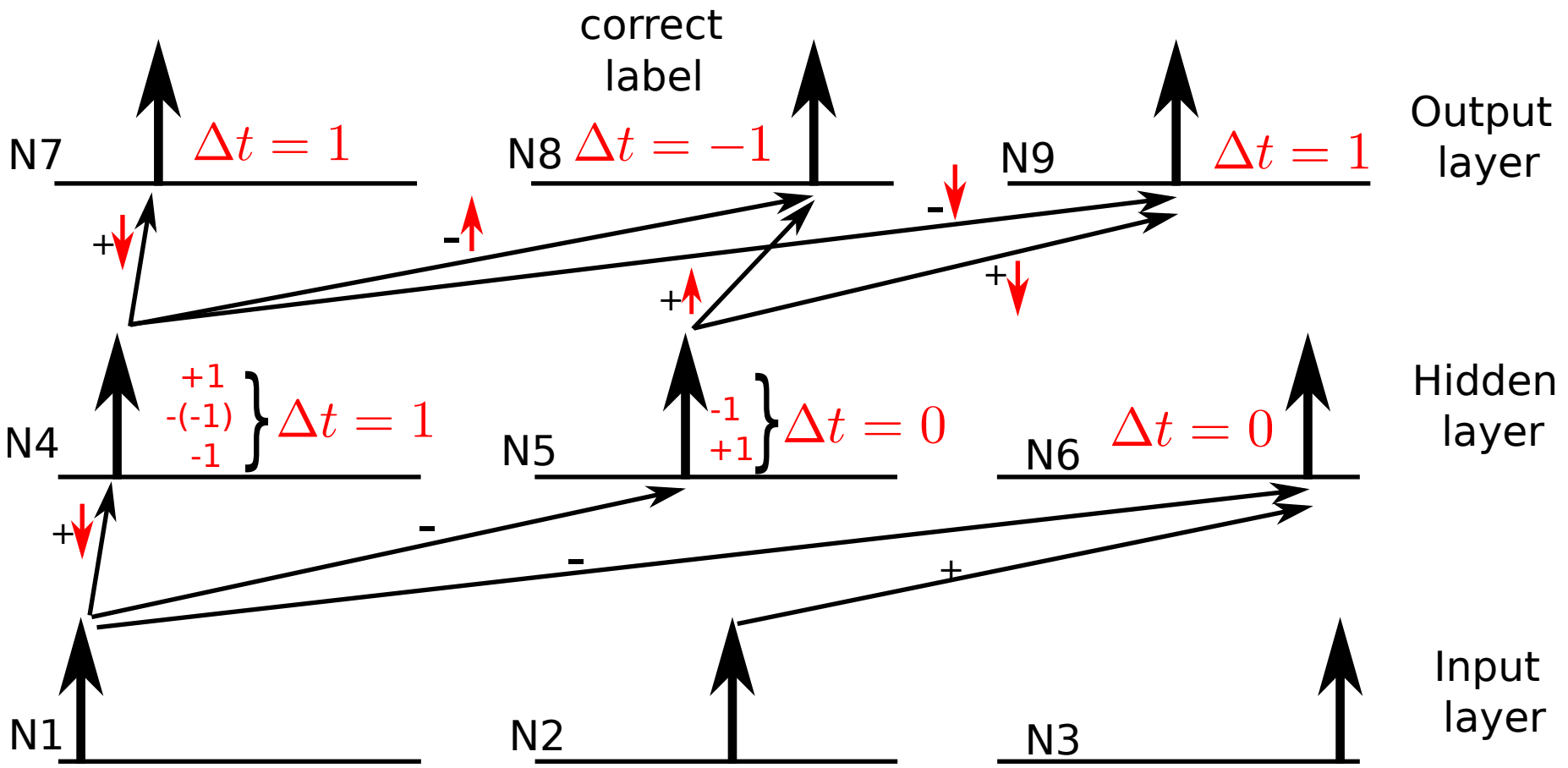
Approximate learning



- Update Hidden→Output weights to encourage the right neuron to spike first
- Only update weights that actually contributed to output timings

Approximate learning



- Backpropagate time deltas using only the sign of the weights
- The final time delta at a hidden layer neuron can be obtained using 2 parallel popcount operations (count 1s in a bit vector) and a comparison.