Inferring Inference

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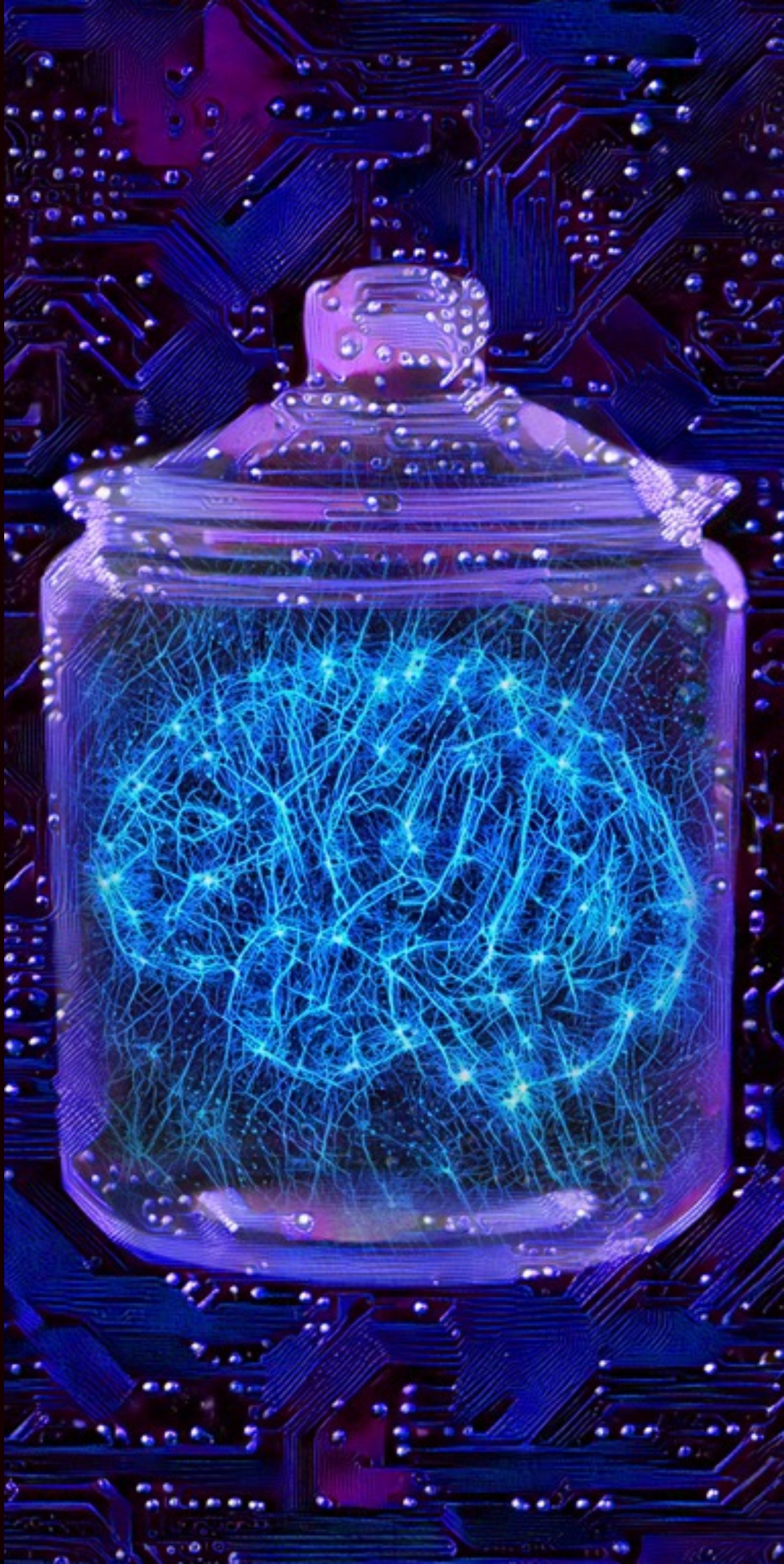
part of the MICrONS project with Tolias, Bethge, Patel, Zemel, Urtasun, Xu, Siapas, Paninski, Baraniuk, Reid, Seung

NICE workshop 2017



World

match

Brain

Hypothesis:

The brain

approximates probabilistic inference

over a probabilistic graphical model

using a message-passing algorithm

implicit in population dynamics

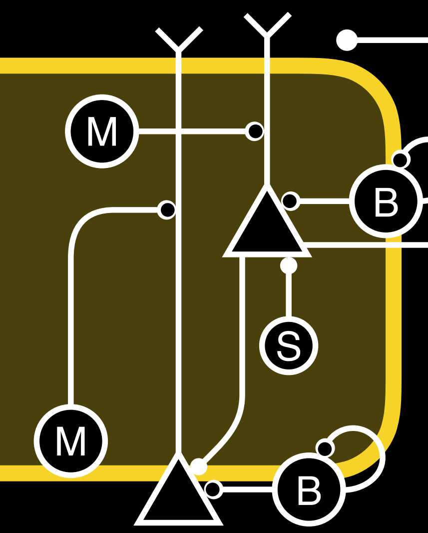
What algorithms can we learn from the brain?

Architectures?

cortex, hippocampus, cerebellum, basal ganglia, …

Transformations?

nonlinear dynamics from population responses



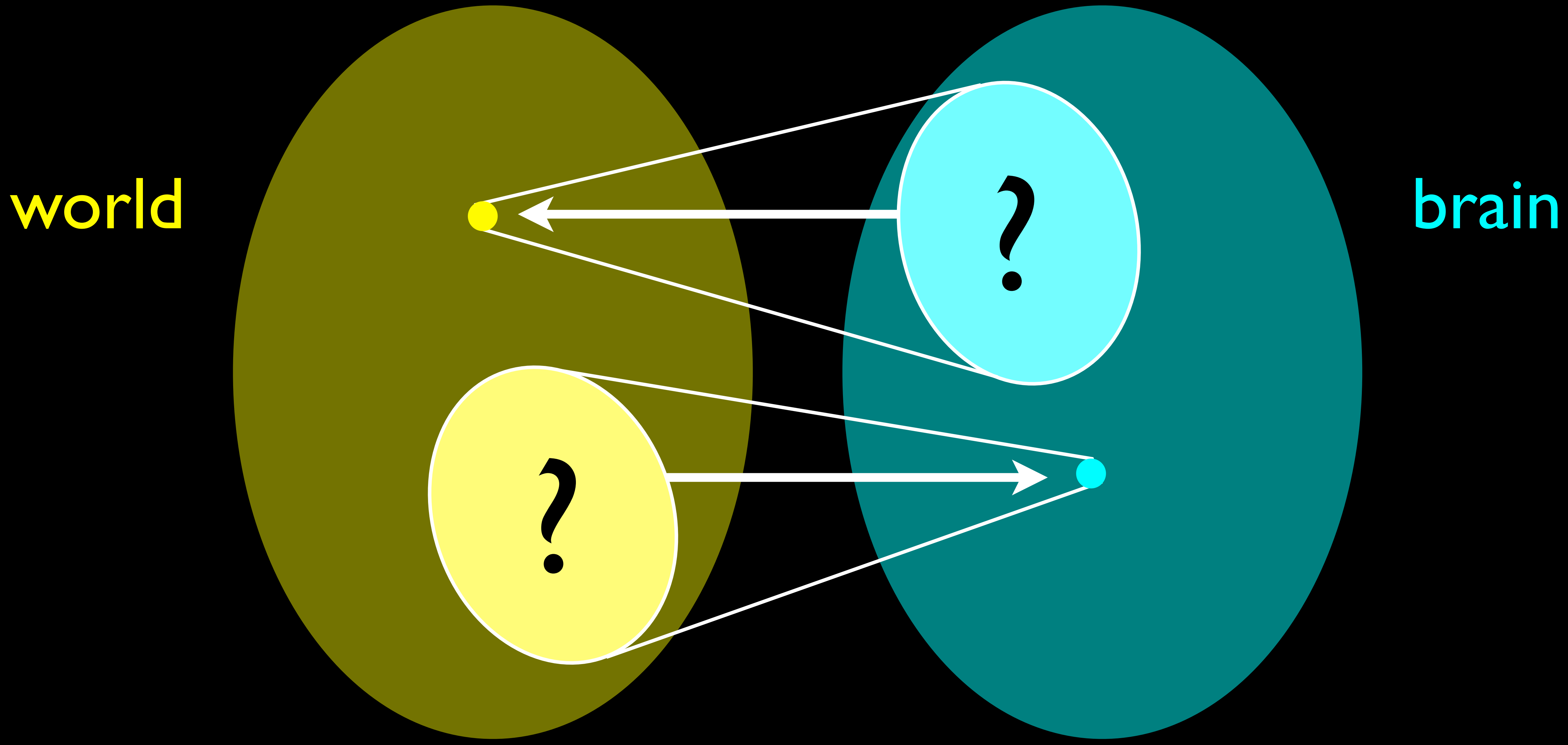
Learning rules?

short and long-term plasticity

Principles:	*Details*:
Probabilistic	Graphical models
Nonlinear	Message-passing inference
Distributed	Multiplexed across neurons

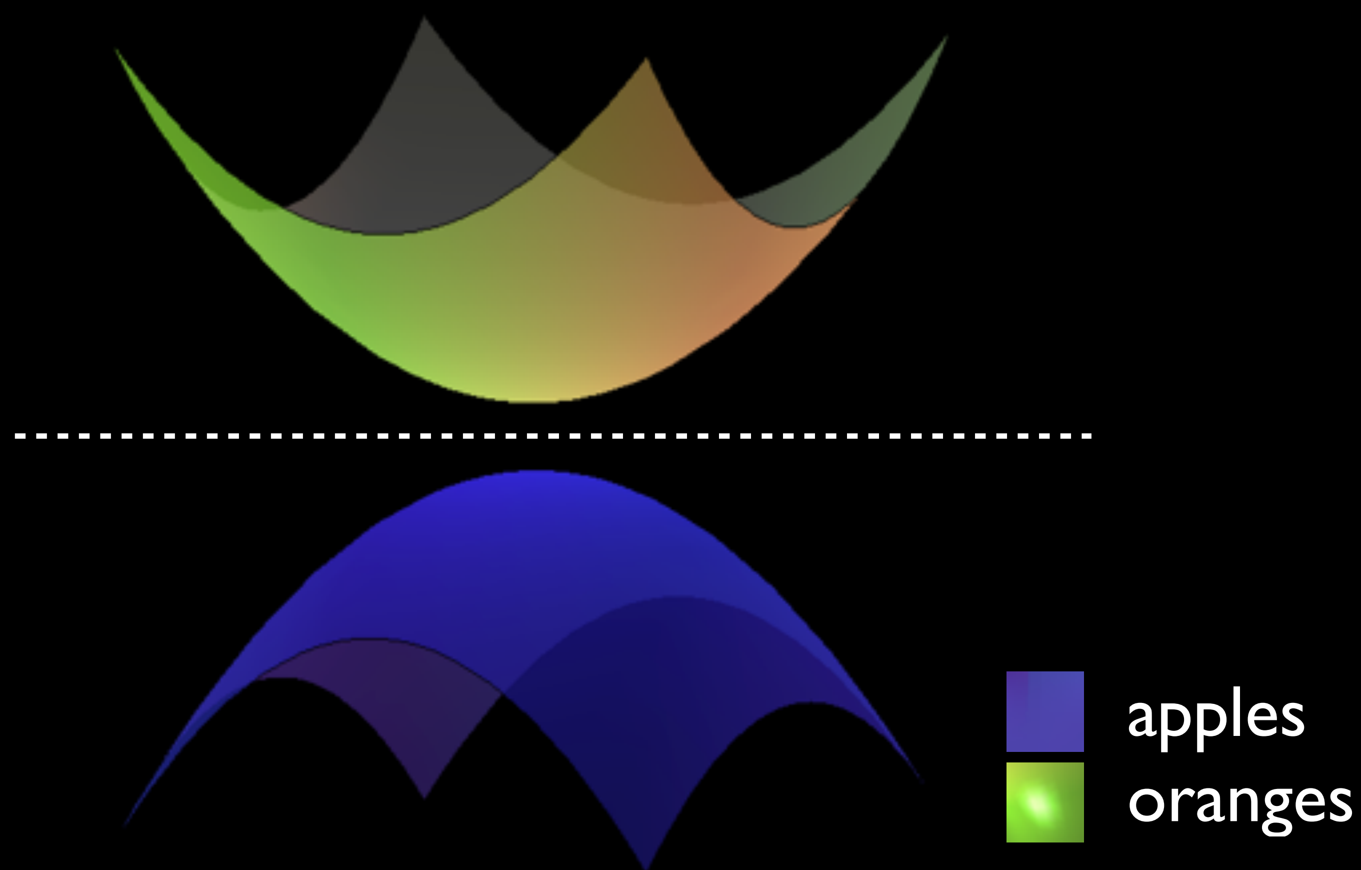
Events in the world can cause many neural responses.
Neural responses can be caused by many events.

So neural computation is inevitably **statistical**.
This provides us with mathematical predictions.

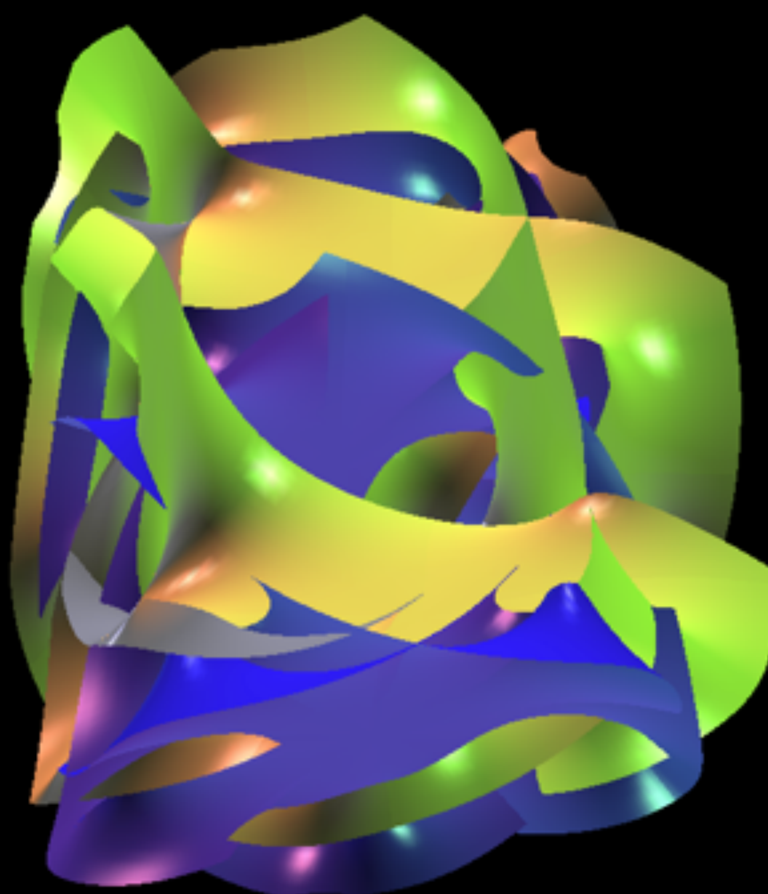


Why does it matter whether processing is linear or nonlinear?

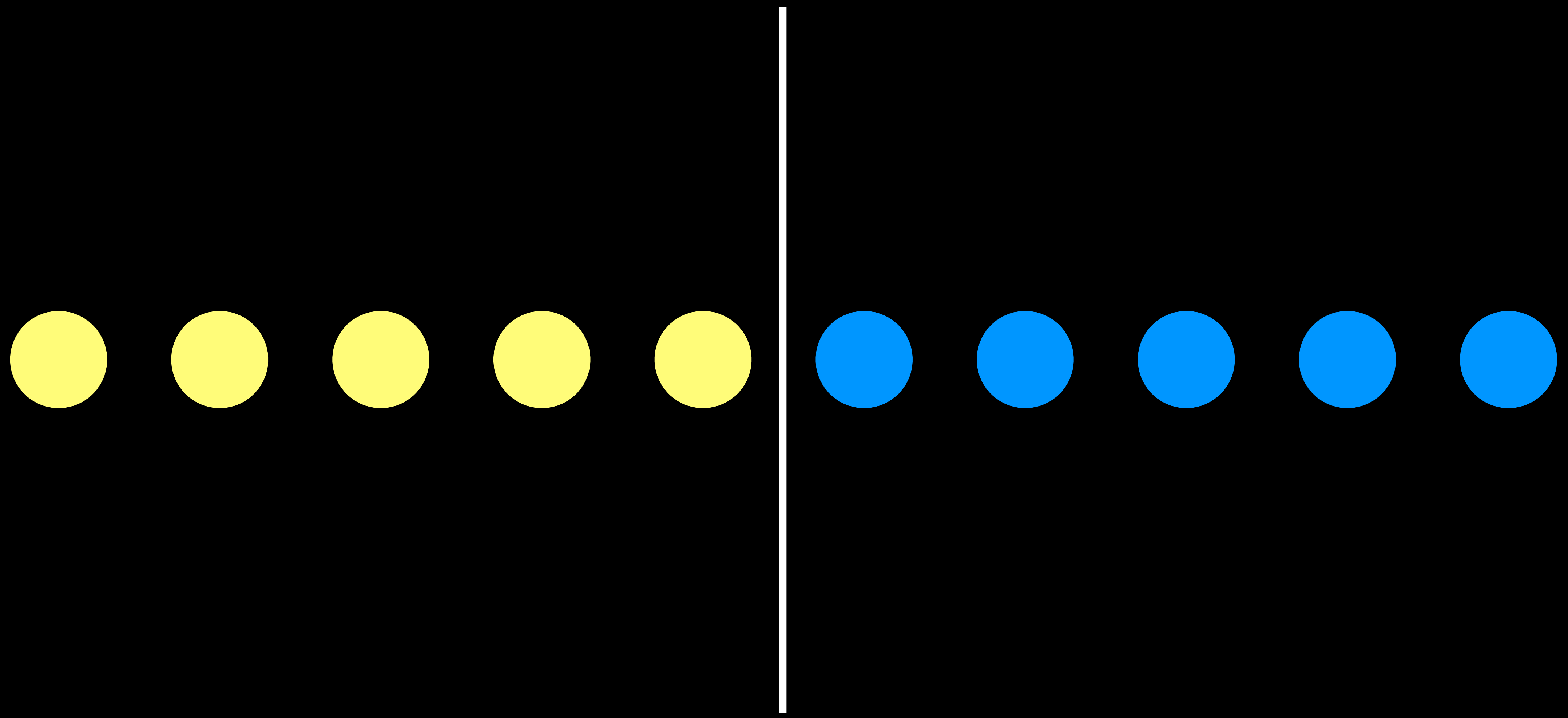
If all computation were linear we wouldn't need a brain.

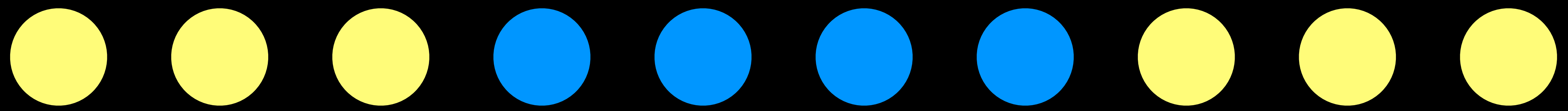


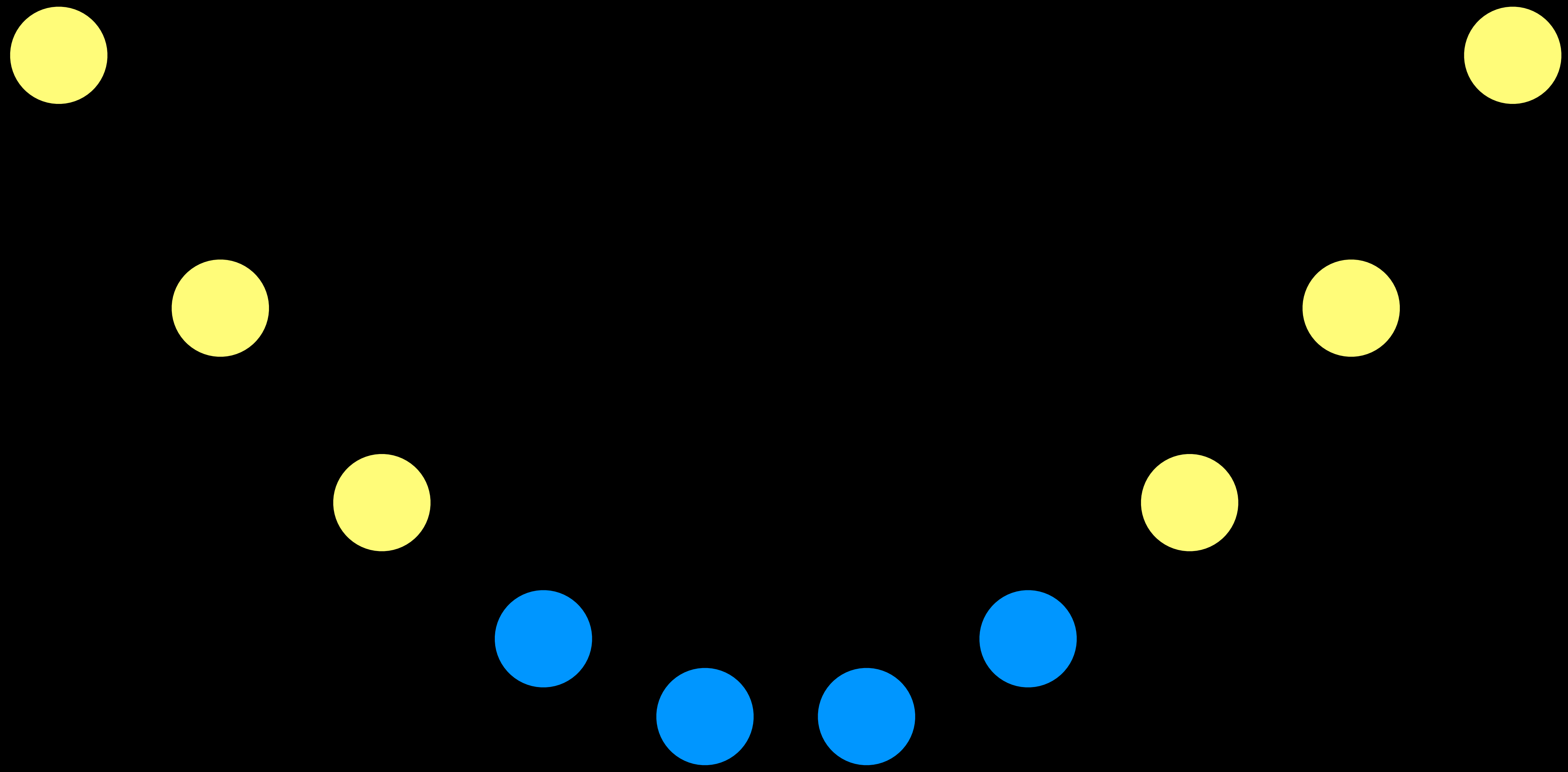
linearly separable

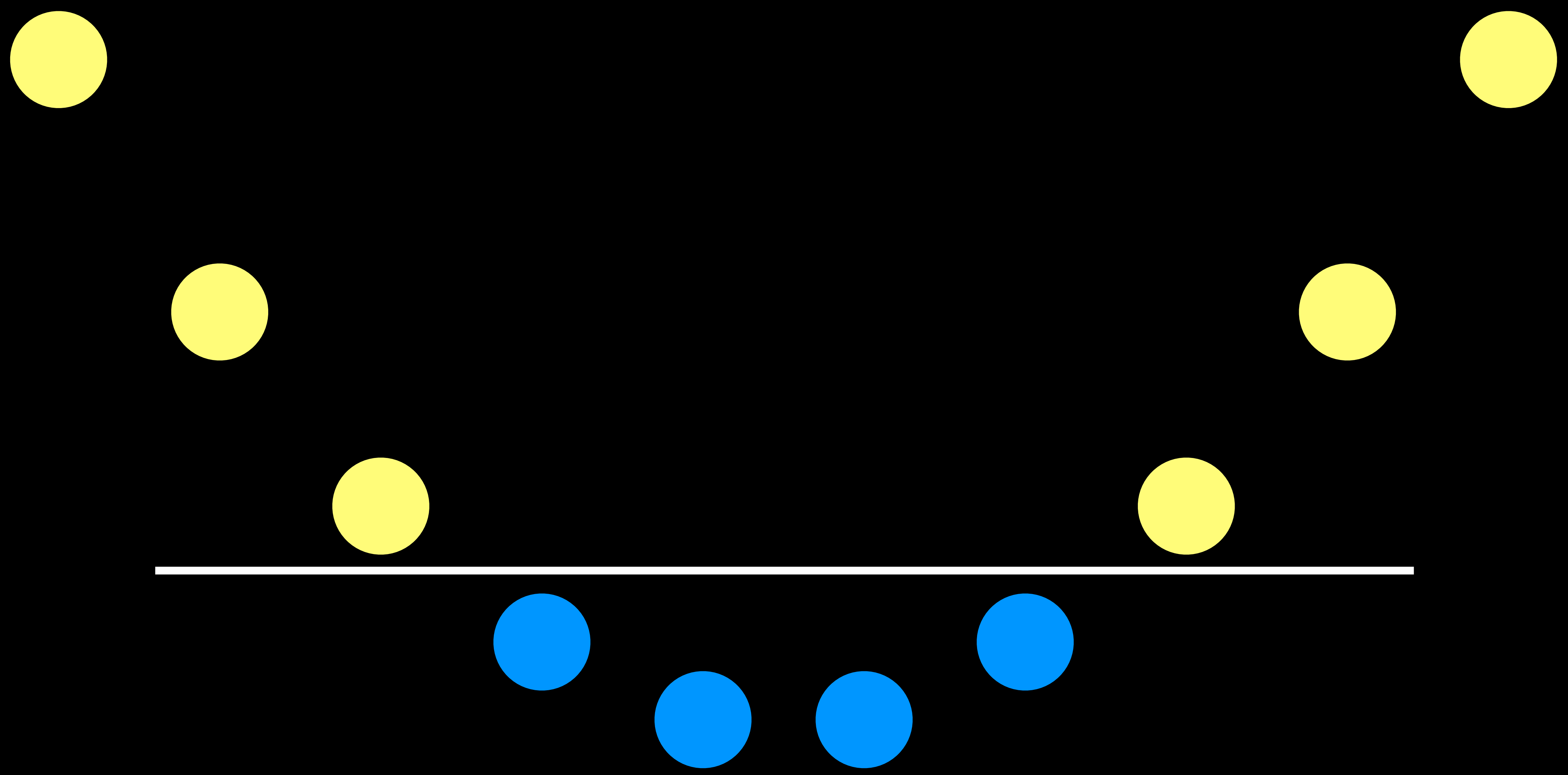


nonlinearly separable





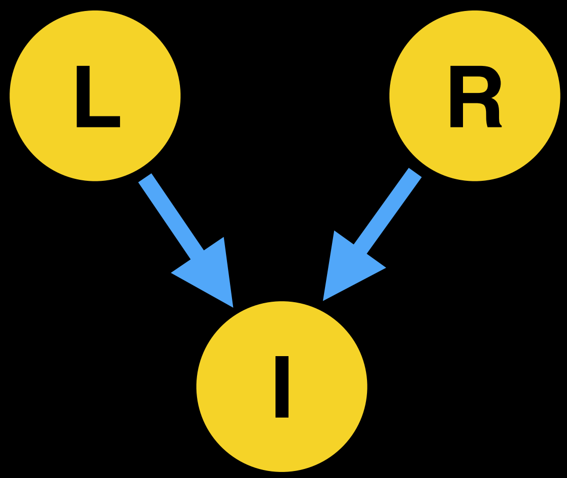




Two sources of nonlinearities

Relationships between latent variables

Image = Light × Reflectance



Relationships between uncertainties

posteriors generally have nonlinear dependencies even for the simplest variables

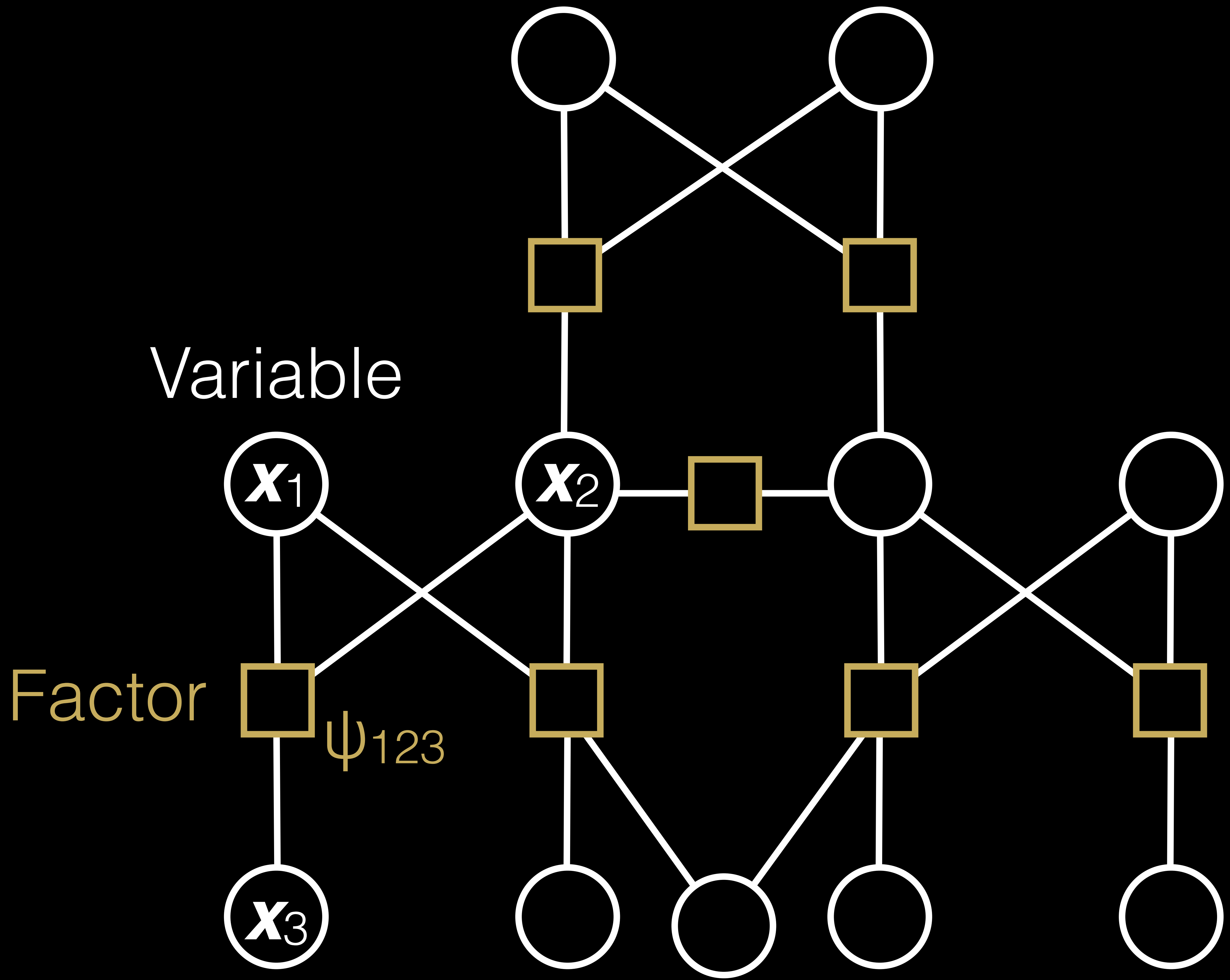
Product rule: $p(x,y) = p(x) \cdot p(y)$

Sum rule: $L(x) = \log \sum_y \exp L(x,y)$

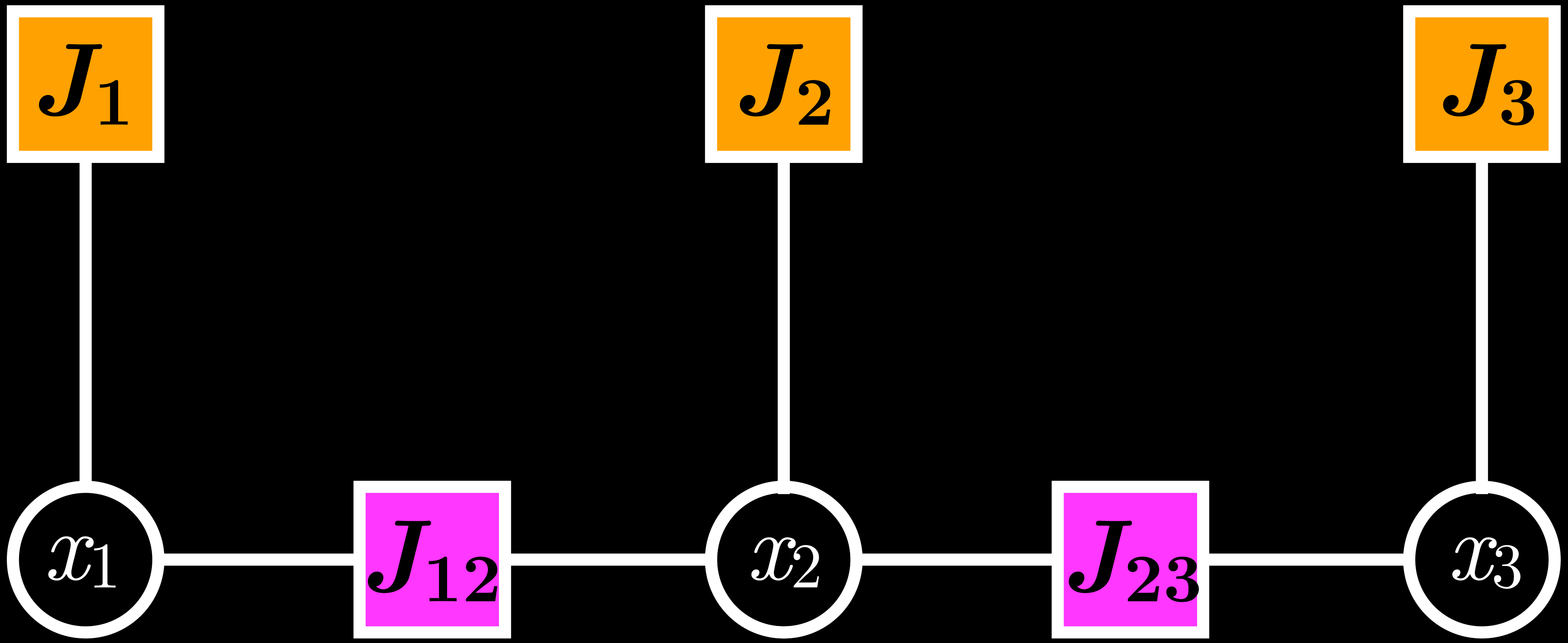
Probabilistic Graphical Models:

Simplify joint distribution $p(\boldsymbol{x}|\boldsymbol{r})$ by specifying how variables interact

$$p(\boldsymbol{x}|\boldsymbol{r}) \propto \prod_{\alpha} \psi_{\alpha}(\boldsymbol{x}_{\alpha})$$


Variable
x_1
x_2
Factor
ψ_{123}
x_3

Example: Pairwise Markov Random Field



$$p(\boldsymbol{x}) = \frac{1}{Z} \prod_{\boldsymbol{s} \in V} e^{\boldsymbol{J}_{\boldsymbol{s}}(\boldsymbol{x}_{\boldsymbol{s}})} \prod_{(\boldsymbol{s},\boldsymbol{t}) \in E} e^{\boldsymbol{J}_{\boldsymbol{st}}(\boldsymbol{x}_{\boldsymbol{st}})}$$

Approximate inference by *message-passing*:

- Localize information so it is actionable
- Summarize statistics relevant for targets
- Send that information *along graph*
- Iteratively update factors with new information

general equation

interactions

$$\boldsymbol{\theta}_{i,t+1} = \boldsymbol{f}(\boldsymbol{\theta}_{it}, \{\boldsymbol{\theta}_{jt}\}_{j \in N_i} | G, J)$$

posterior parameters

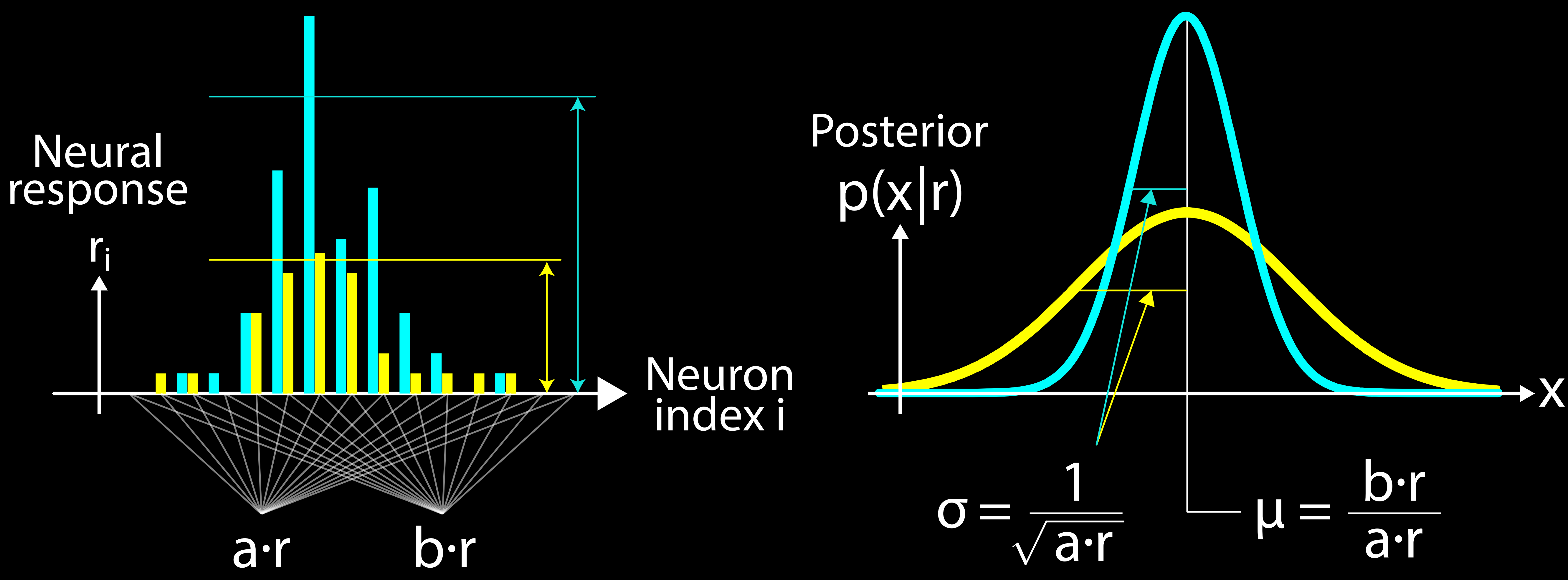
posterior for neighbors

message-passing parameters

Example message-passing algorithms

- Mean-field (assumes variables are independent)
- Belief propagation (assumes tree graph)
- Expectation propagation (updates parametric posterior)
- …
- Brain's clever tricks?

Spatial representation of uncertainty
(e.g. Probabilistic Population Codes, PPCs)



Pattern of activity represents probability.

More spikes generally means more certainty

Ma, Beck, Latham, Pouget 2006, etc

Message-passing updates

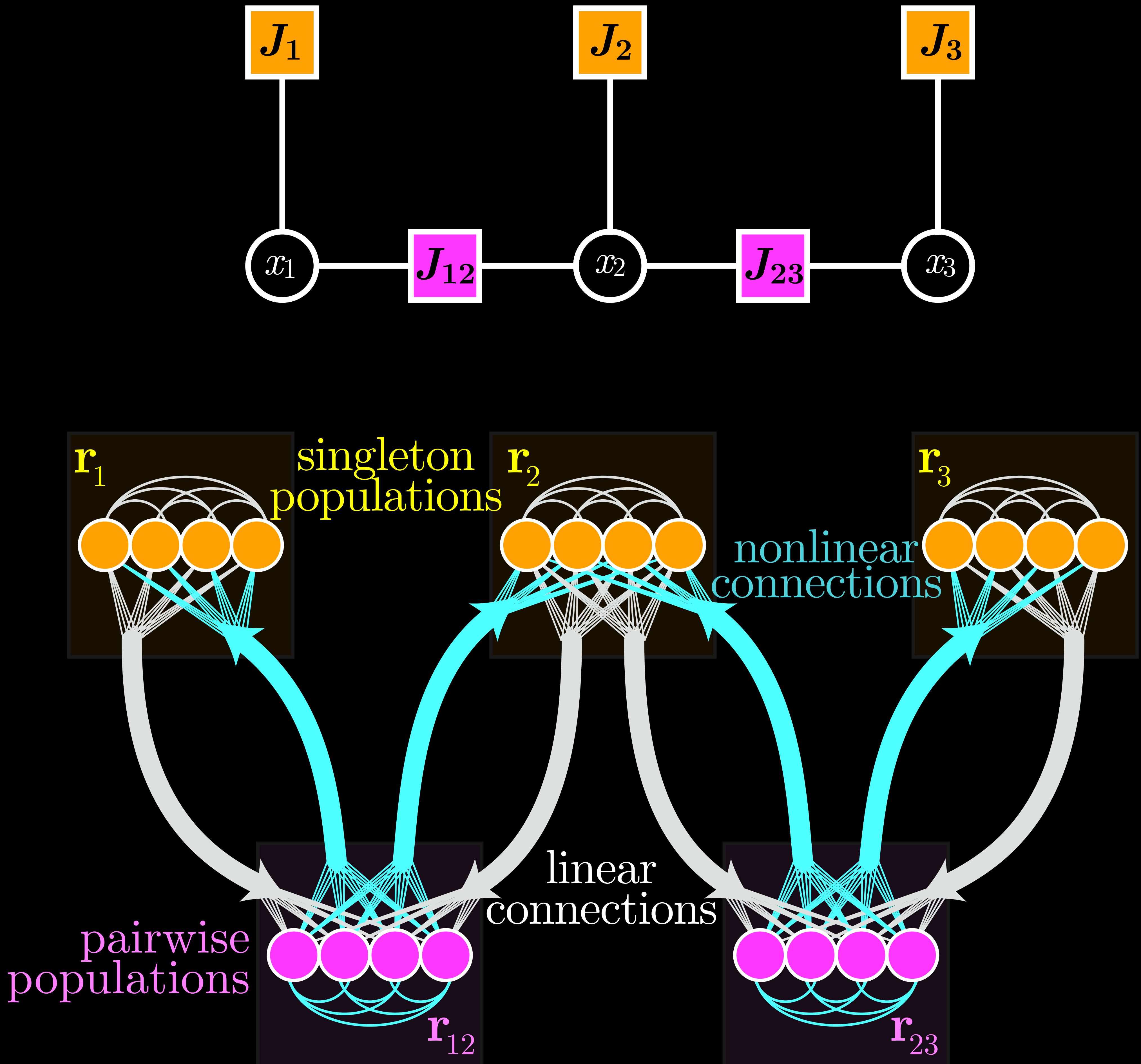
$$\boldsymbol{\theta}_{i,t+1} = \boldsymbol{f}(\boldsymbol{\theta}_{it}, \{\boldsymbol{\theta}_{jt}\}_{j \in N_i} | G, J)$$

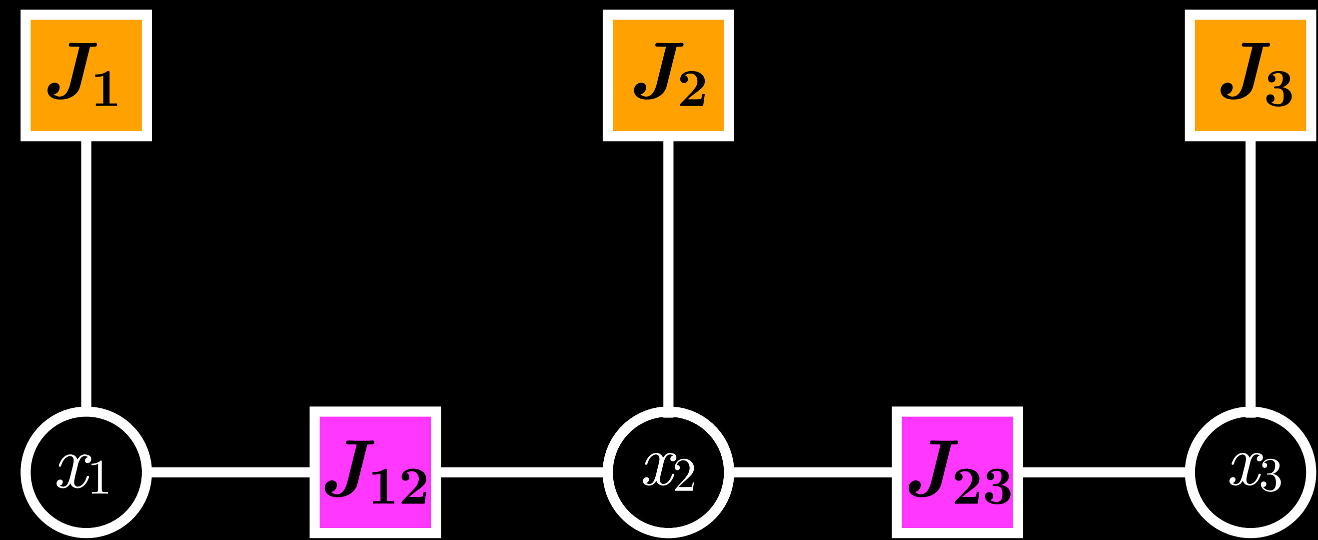
$$\boldsymbol{r} = U\boldsymbol{\Theta} + \boldsymbol{\eta}$$

embedding

$$\dot{\boldsymbol{r}} = \boldsymbol{F}(\boldsymbol{r}_t)$$

Neural dynamics


J_1
J_2
J_3
x_1
J_{12}
x_2
J_{23}
x_3
$\mathbf{r}_1$
singleton populations
$\mathbf{r}_2$
$\mathbf{r}_3$
nonlinear connections
linear connections
pairwise populations
$\mathbf{r}_{12}$
$\mathbf{r}_{23}$


J_1
J_2
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x_1
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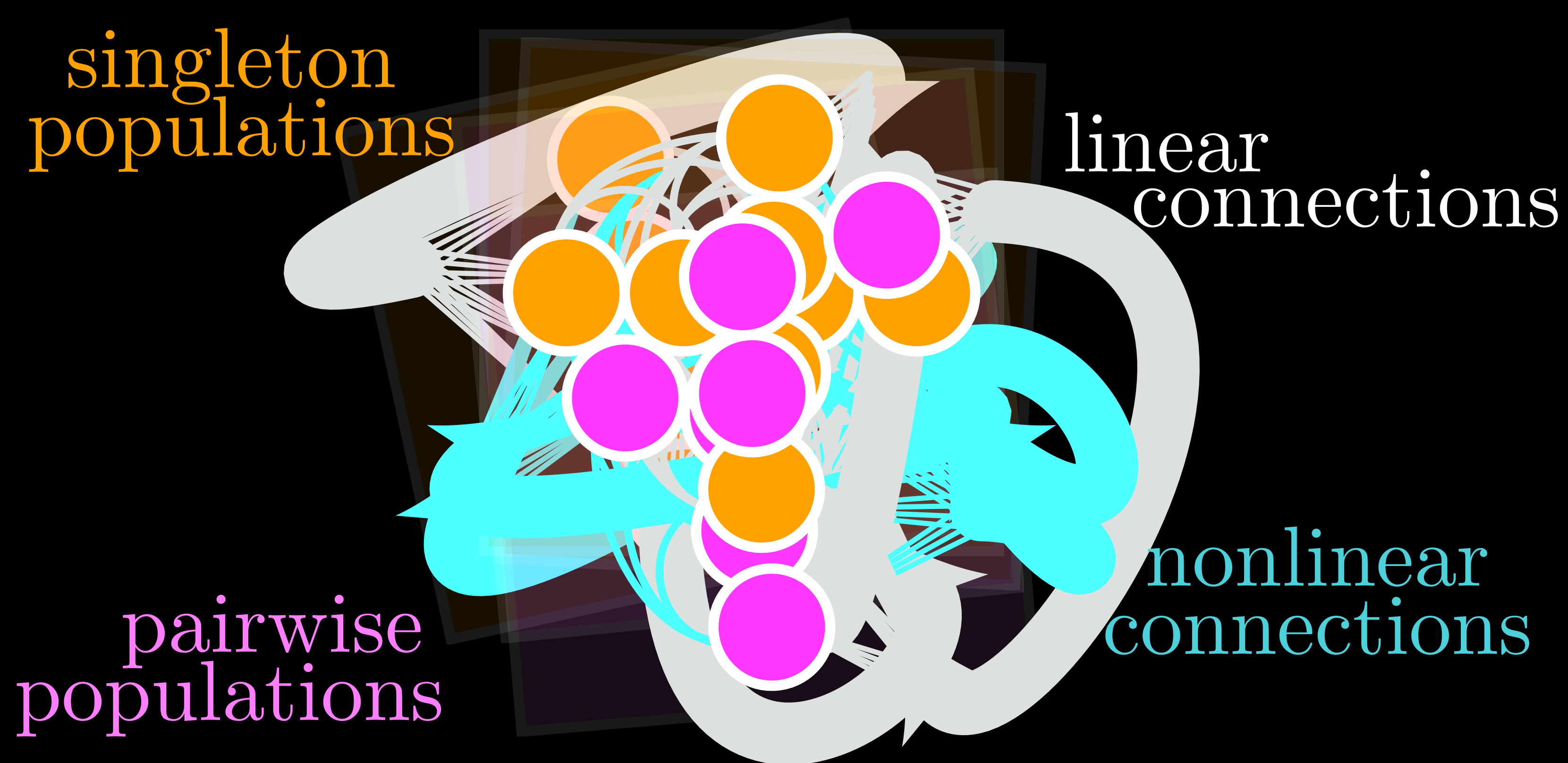
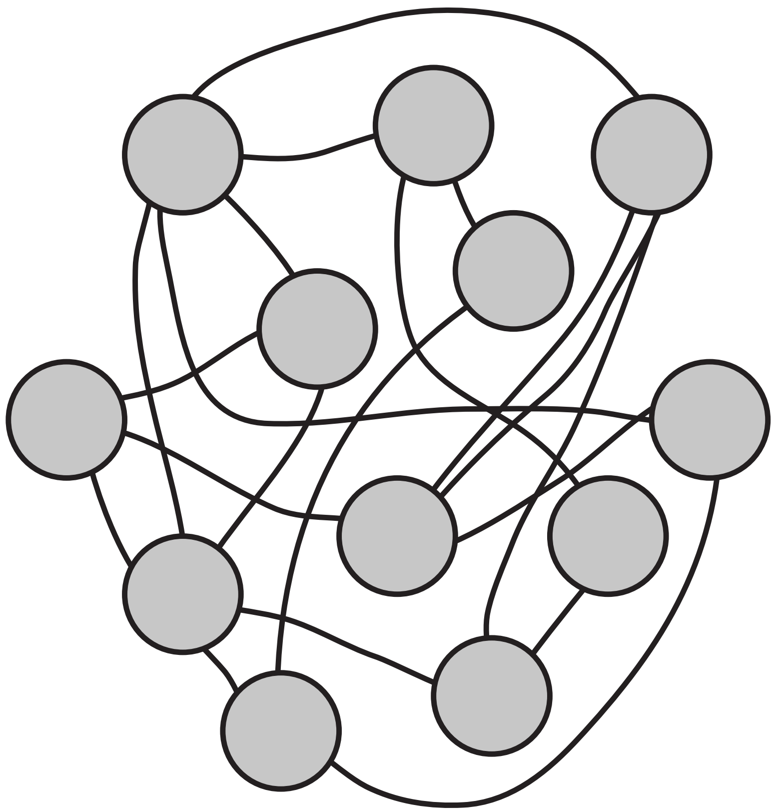
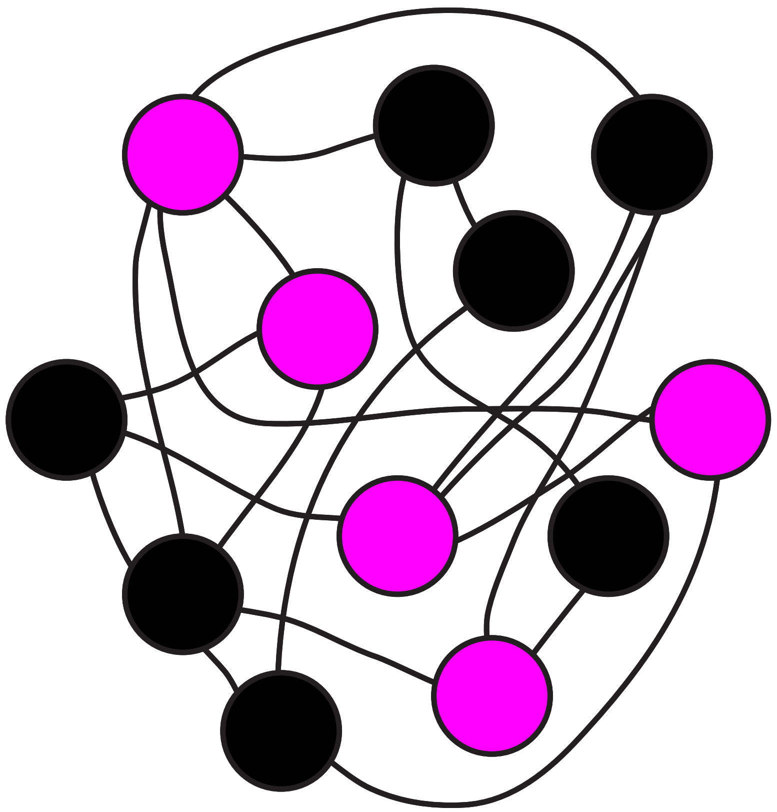
x_2
J_{23}
x_3

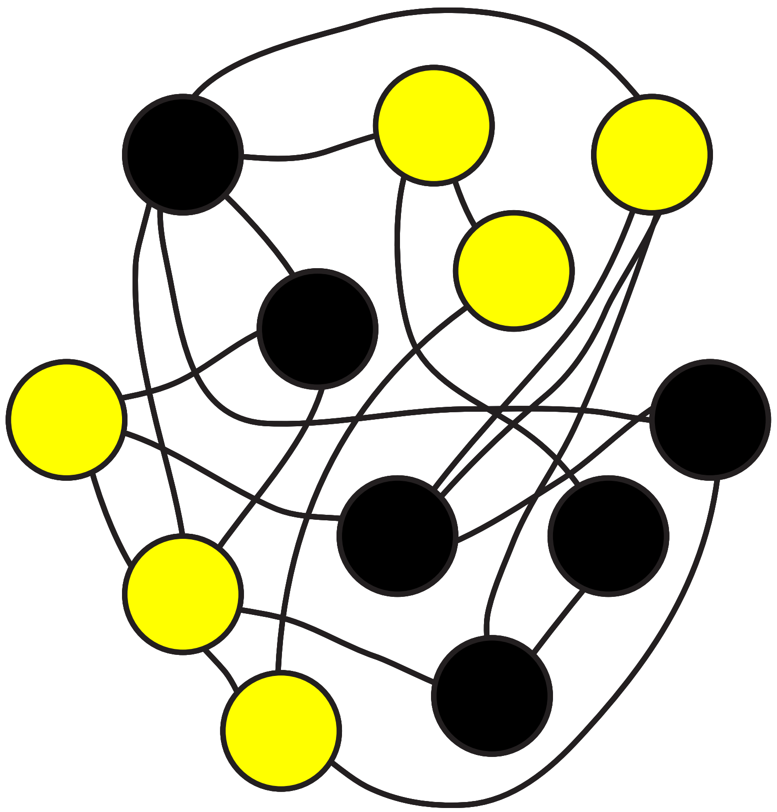

singleton populations
linear connections
nonlinear connections
pairwise populations

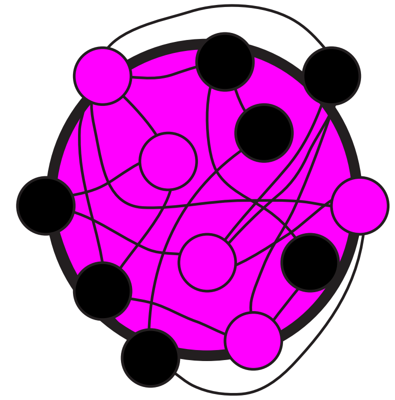


Neural
activity



Neural
activity

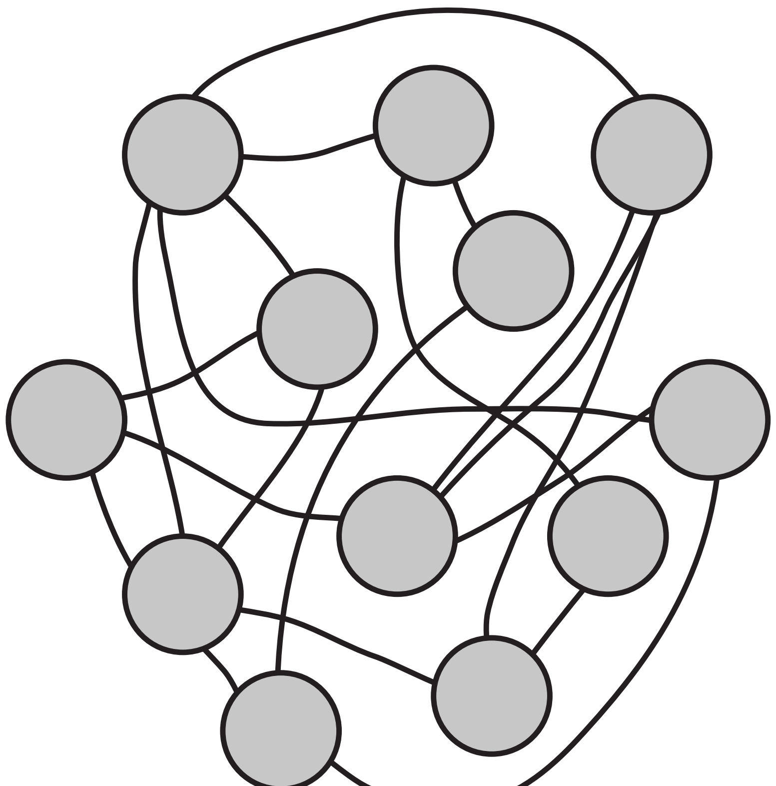


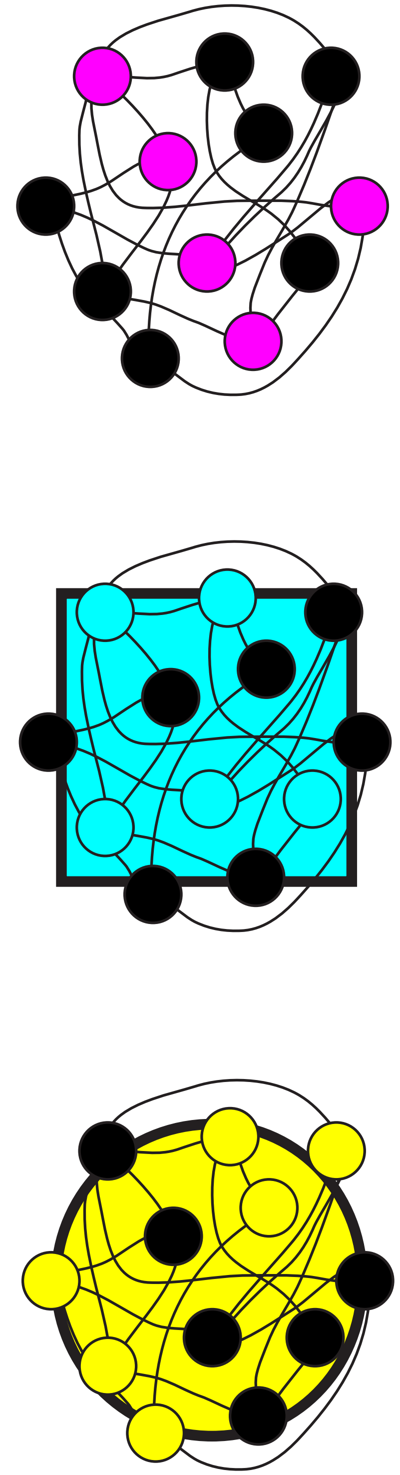


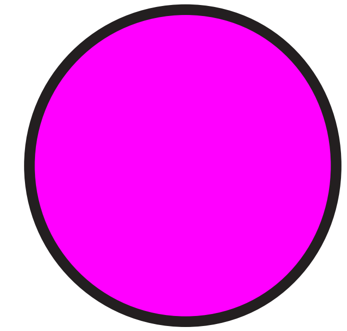
Neural activity

Neural encoding

Information encoded



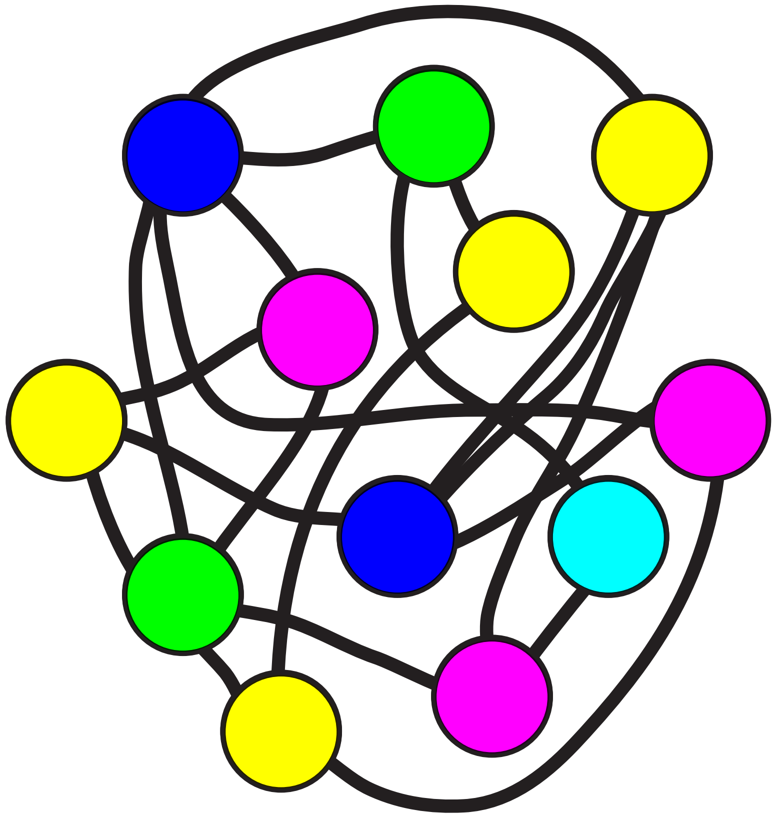




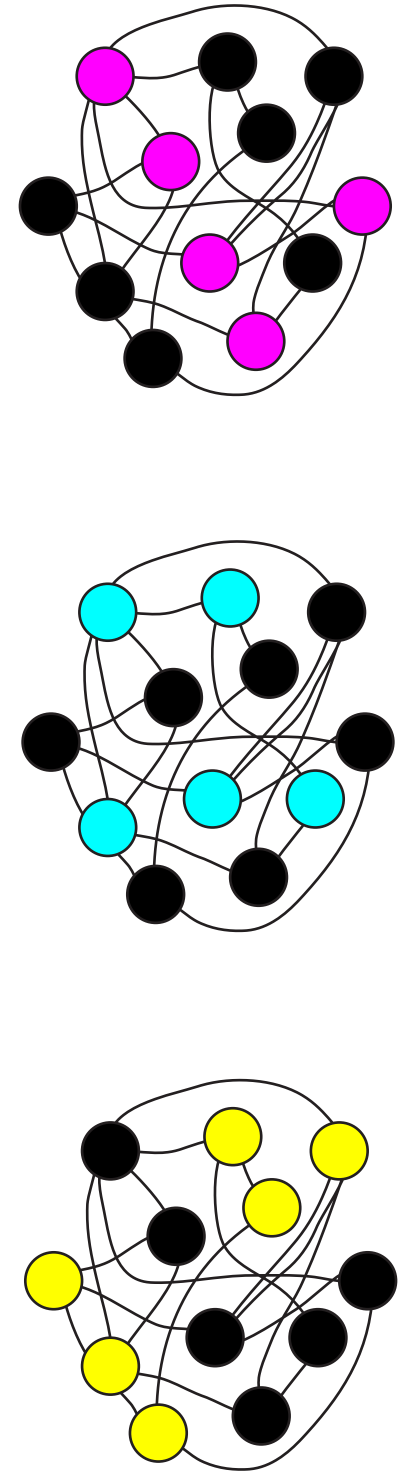
Neural activity

Neural encoding

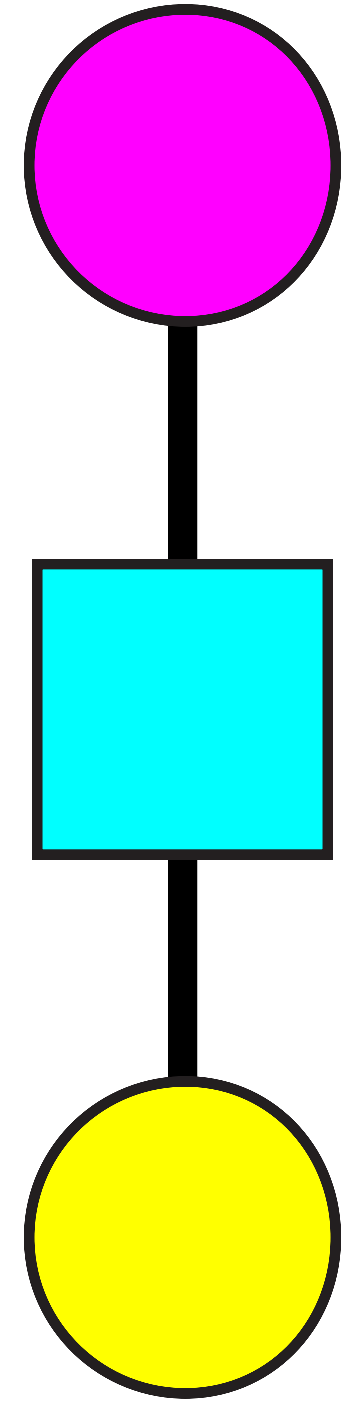
Information encoded



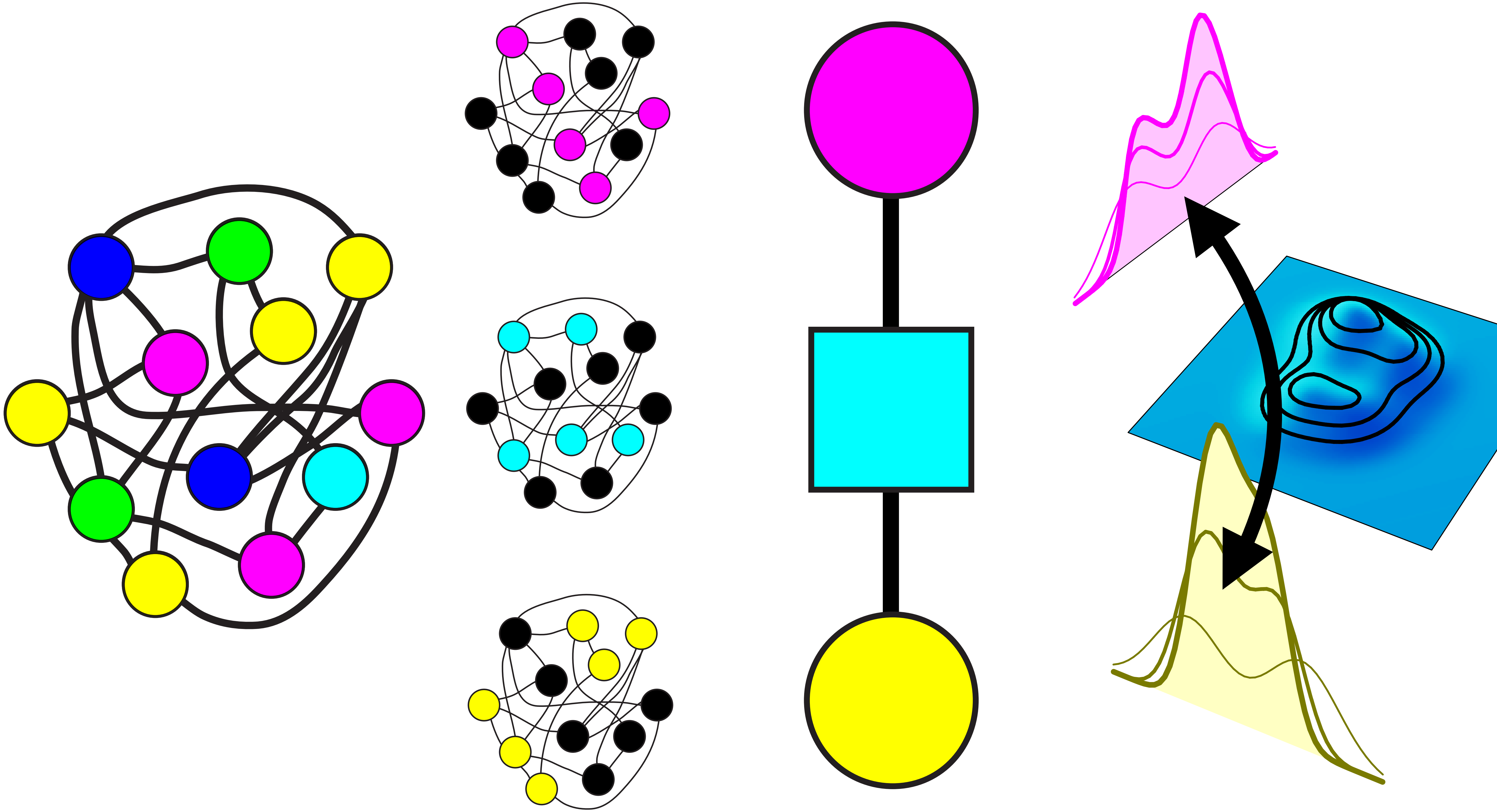
Neural **interactions**



Neural encoding



Information **interactions**

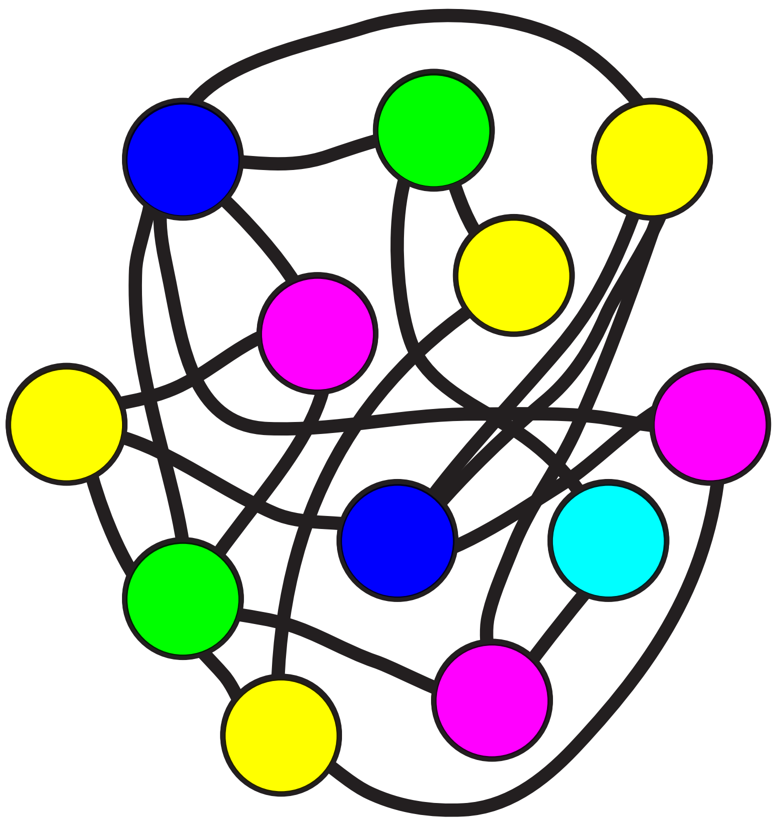


Neural **interactions**

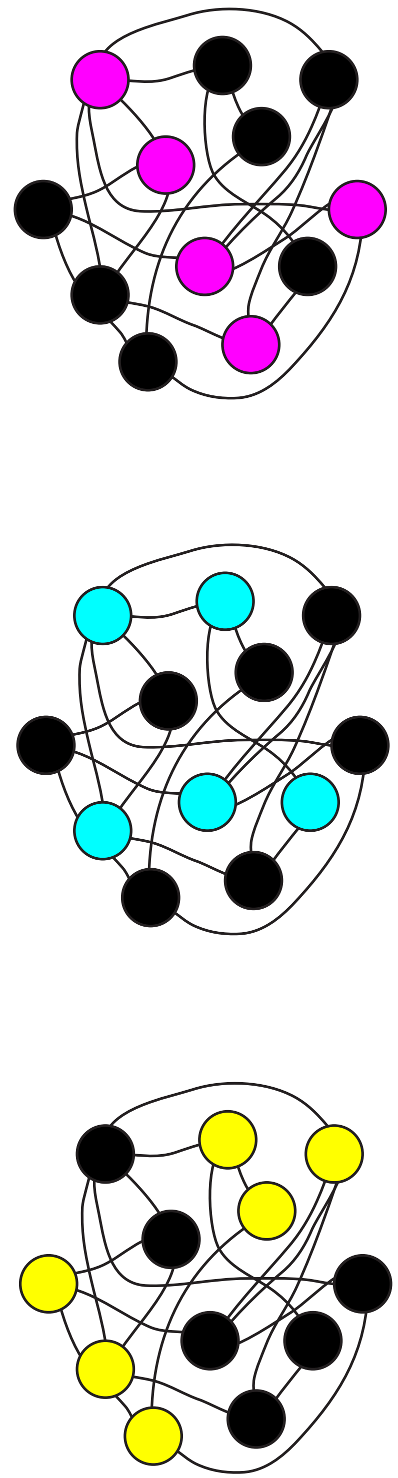
Neural encoding

Information **interactions**

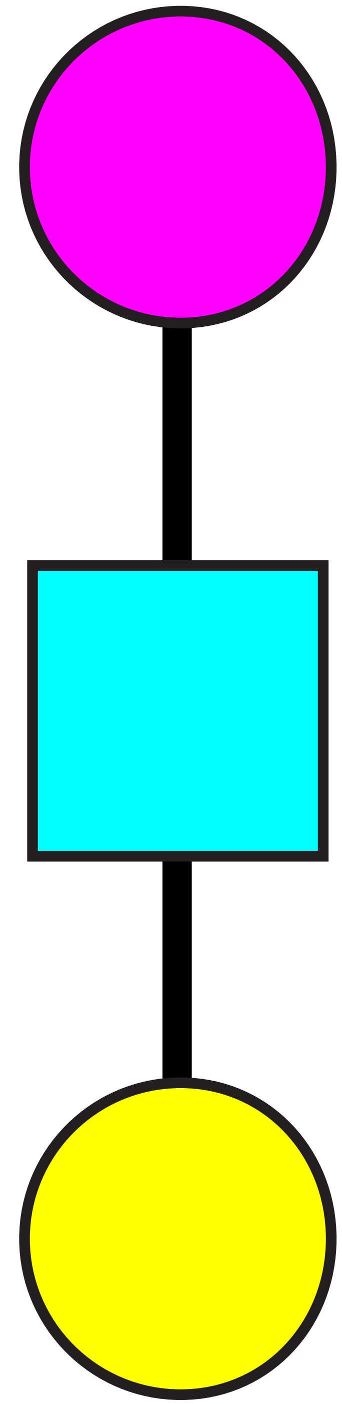
Probability distributions



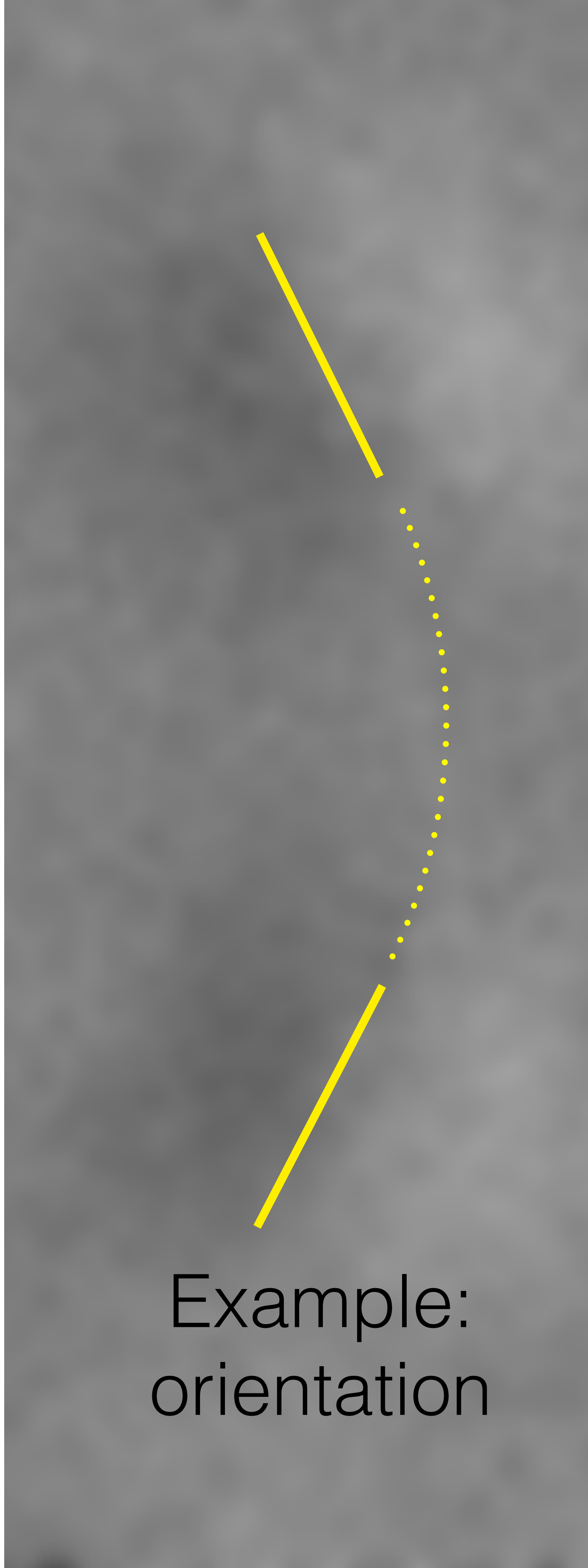
Neural
interactions



Neural
encoding

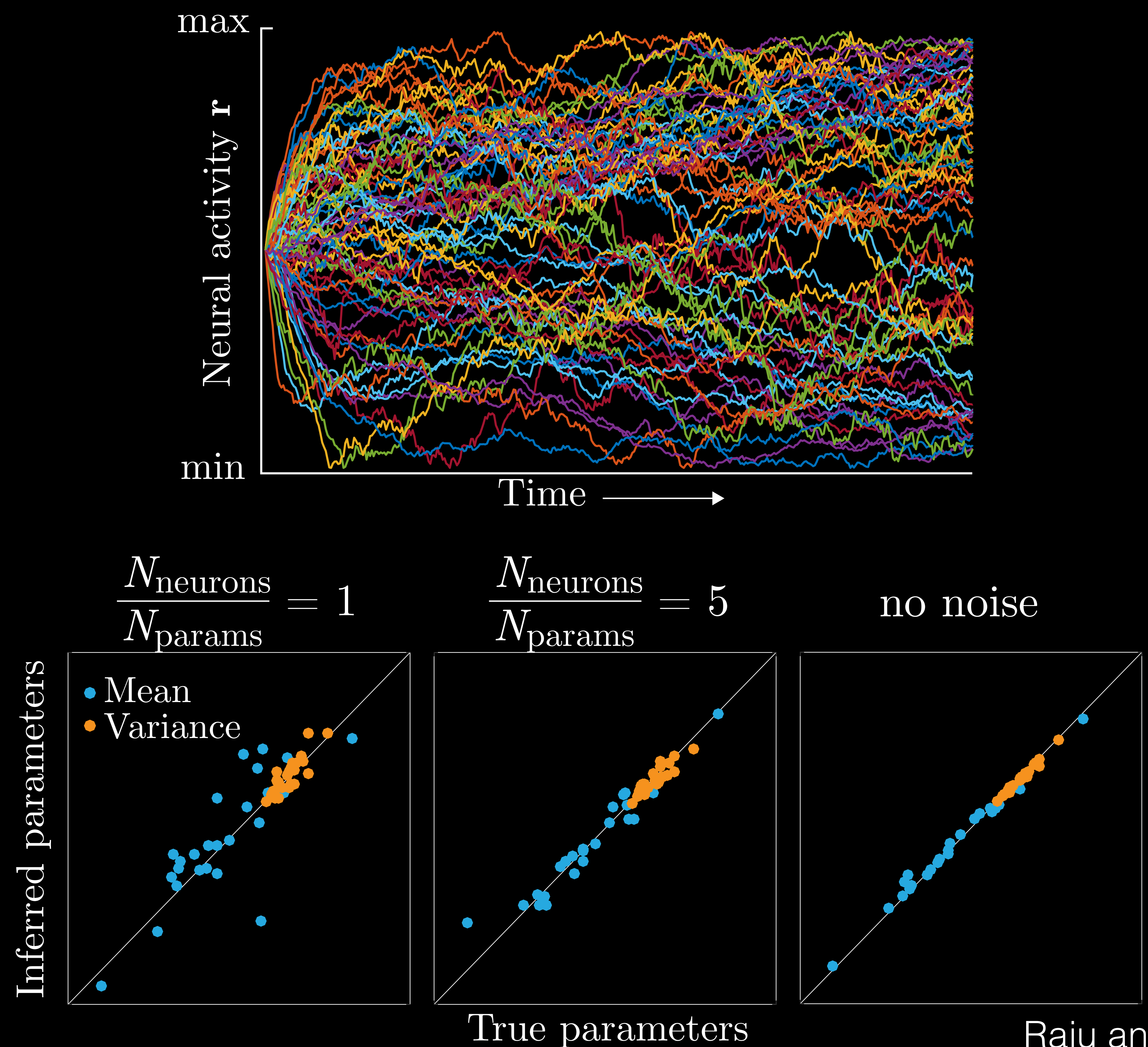


Information
interactions



Example:
orientation

Network activity can implicitly perform inference



Raju and Pitkow 2016

Simulated brain

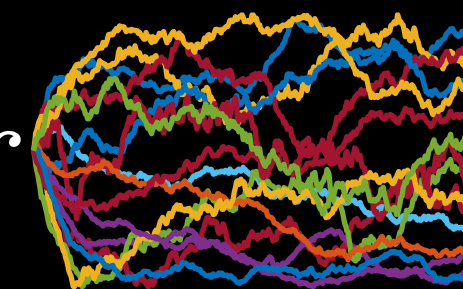
$$\dot{b}_{it} = -b_{it} + \sigma\left(\sum_j W\left[J_{ij}, b_{it}, b_{jt}\right] b_{jt} + h_{it}\right)$$

$$W\left[J_{ij}, b_i, b_j\right] = 2J_{ij} + 4J_{ij}^2(1-2b_i)(1-b_j)$$

Infer

$$\boldsymbol{r}_{t+1} = \sigma\left(A\boldsymbol{r}_t + B\boldsymbol{h}_t - \theta\right)$$

Encode



Inferring inference

$$\hat{\boldsymbol{b}} = V\boldsymbol{r} + c$$

Decode

$$\hat{W}[J_{ij}, b_i, b_j] = \sum_{\alpha\beta\gamma} \underline{G_{\alpha\beta\gamma}}\ \underline{J_{ij}^{\alpha}}\ b_i^{\beta}\ b_j^{\gamma}$$

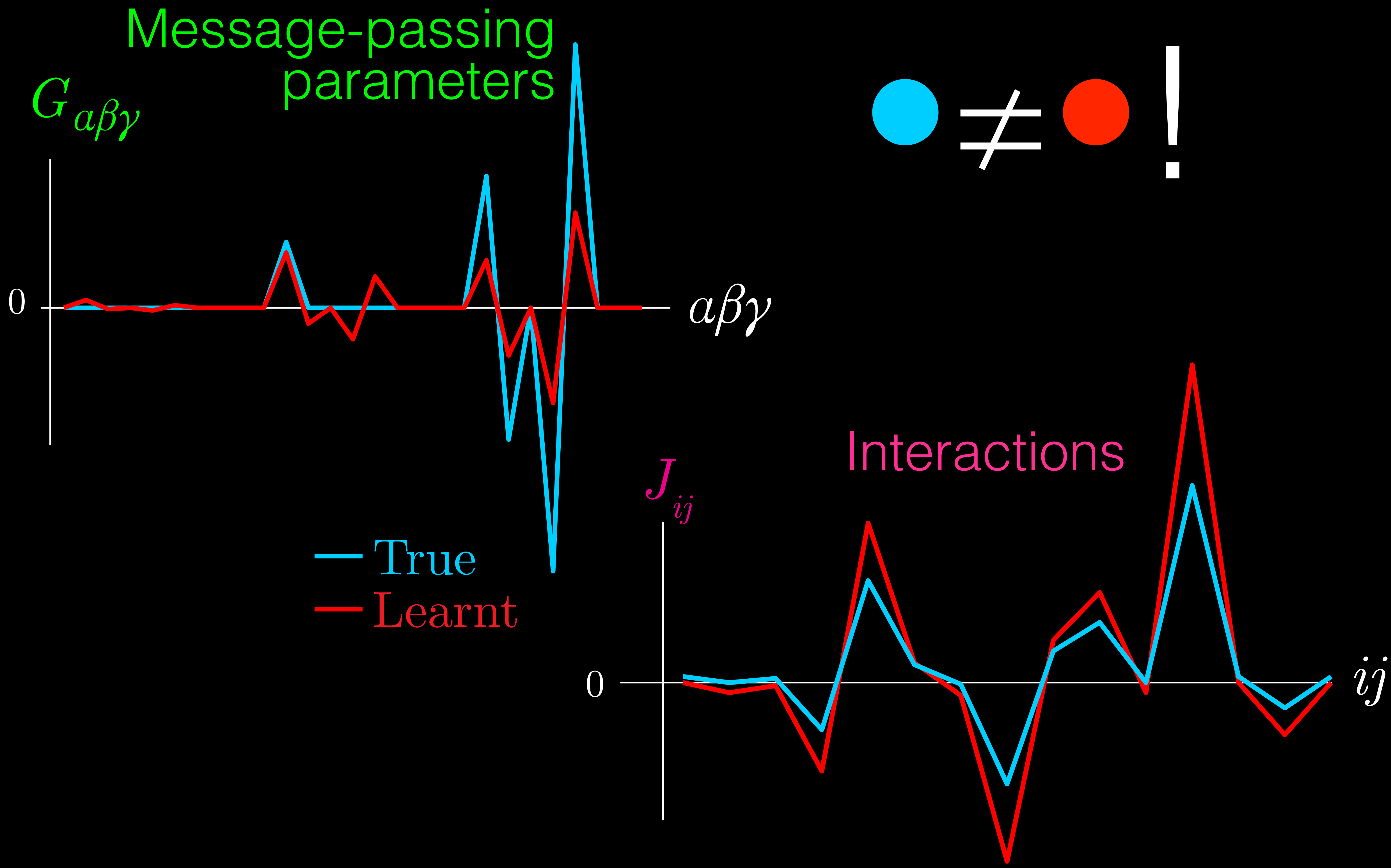
Message-passing parameters

Interactions

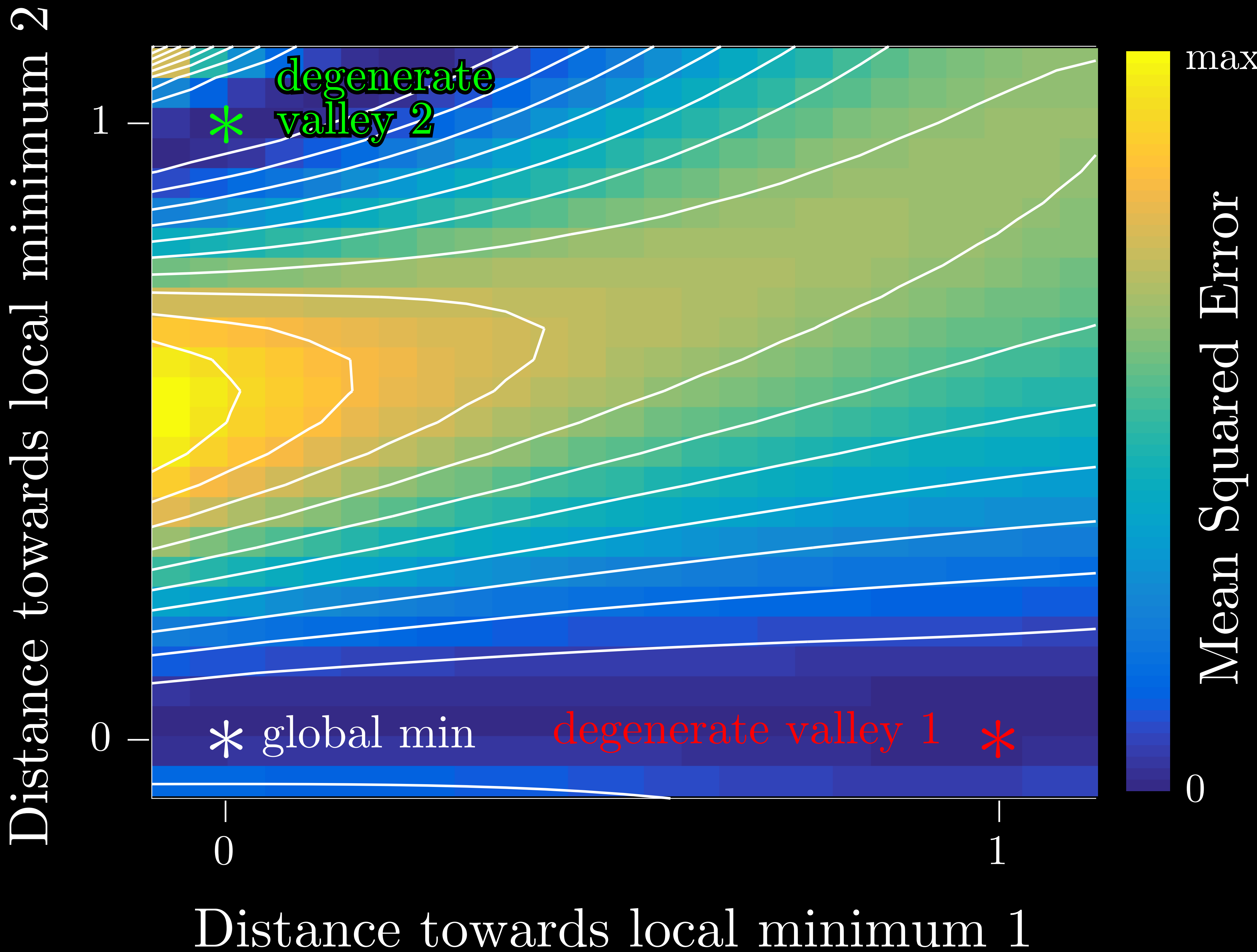
Fit*

*within family

Recovery results for simulated brain



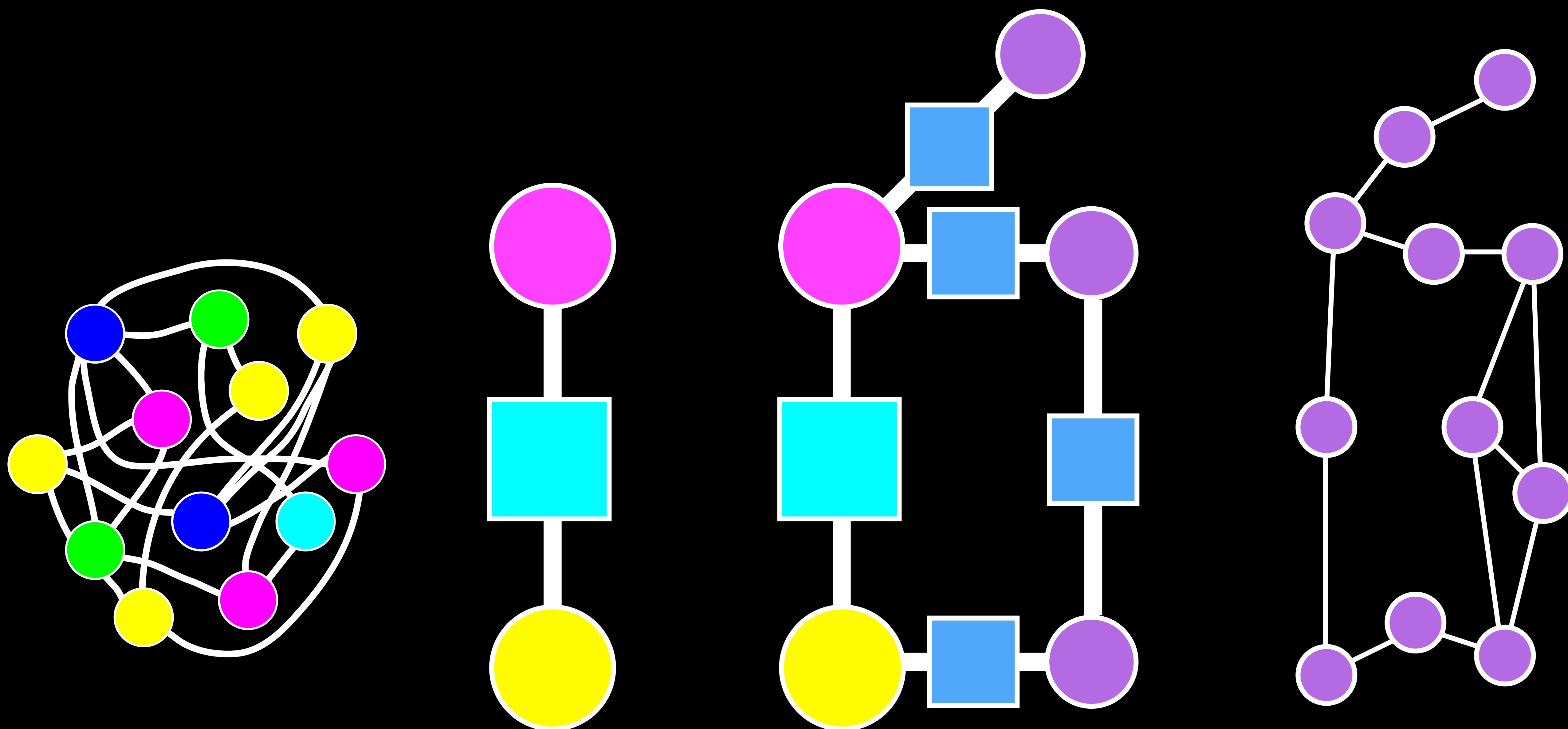
Analysis reveals degenerate family of *equivalent* algorithms



From *simulated* neural data we have recovered:

how variables are encoded	Representation
which variables interact how they interact	Graphical model
how the interactions are *used*	***Message-Passing algorithm***

Applying message-passing to novel tasks



Brain neural network

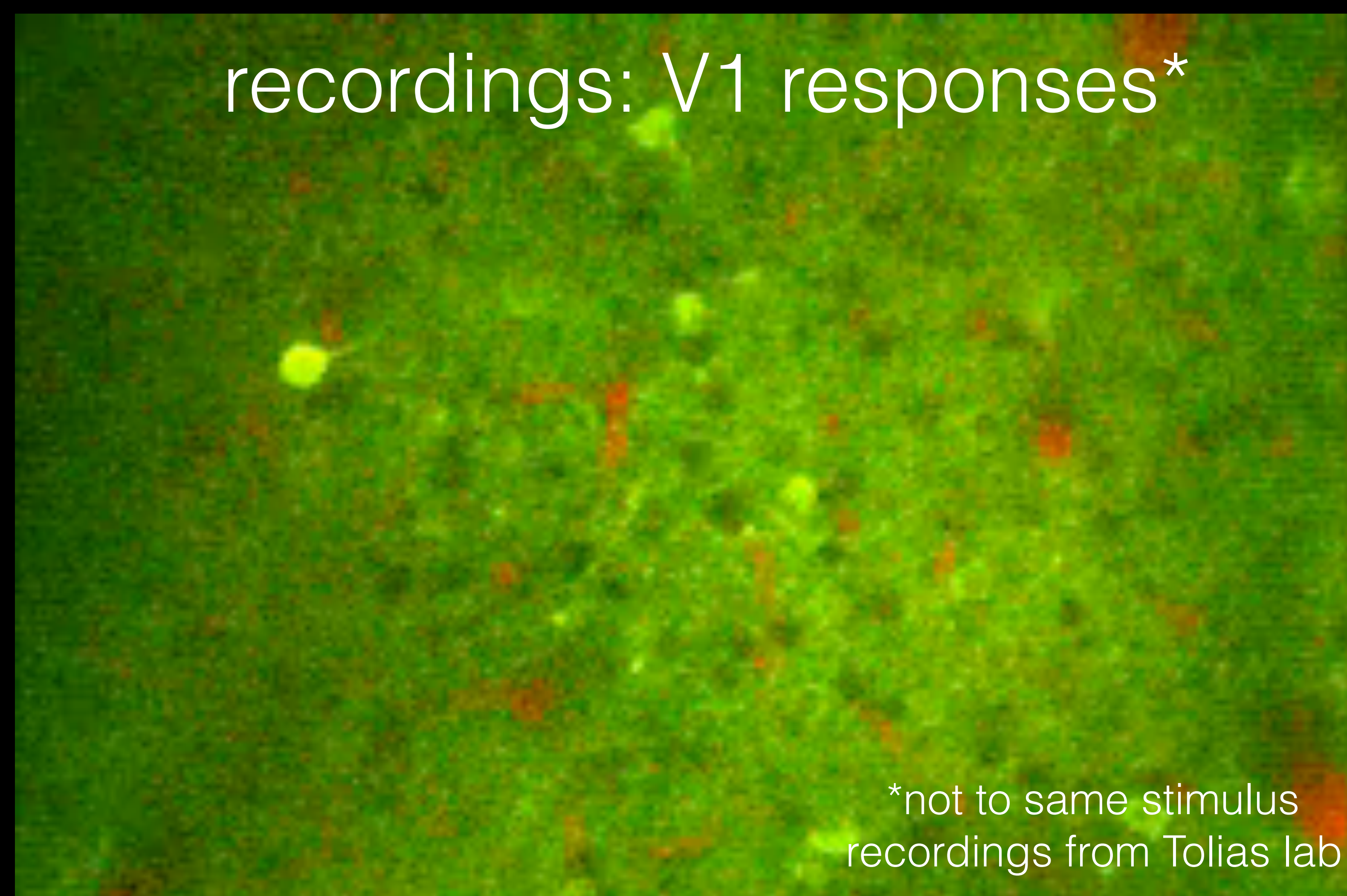
Message passing nonlinearity

Apply to new graphical model structure

OR

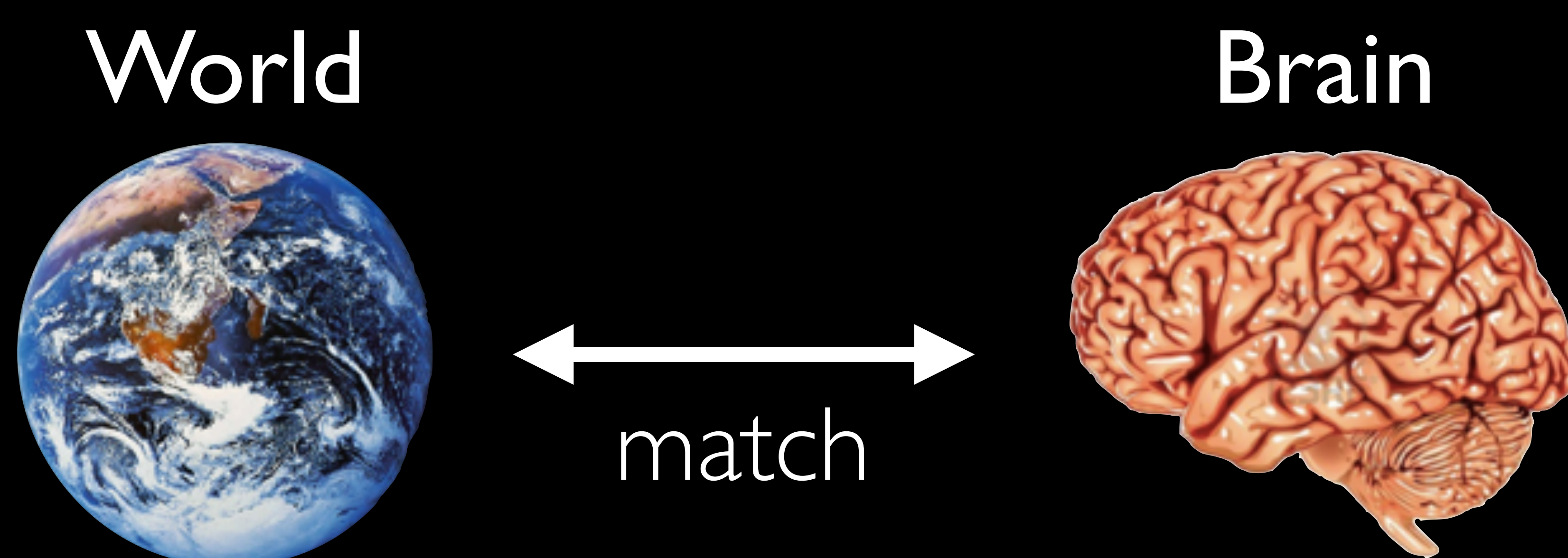
Relax to novel neural network

**Next up:
applying methods
to real brains**



mementos:

- **Neurons can perform inference** ***implicitly*** in a graphical model distributed across a population.
- **New method to discover message-passing algorithms** by modeling transformations of decoded task variables



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